Day 93 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{8}{15}$. It moves 4.9 m in 2 s. Find (a) the coefficient of friction, (b) the speed after 2 s. Take $g = 9.8$.

$s = \frac{1}{2}at^2$: $4.9 = \frac{1}{2}\times a\times 2^2$, so $a = 2.45$

Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$

$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8\left(\frac{8}{17} - \frac{15}{17}\mu\right) = 2.45$

$\frac{8}{17} - \frac{15}{17}\mu = 0.25$, so $\mu = 0.25$

$v = at = 2.45\times 2$

Answer: $\mu = 0.25$; 4.9 m s^{-1}

2 Change of sign to locate a root (All boards)

$f(x) = x^2 - 2\cos x - 12$.

Show that the equation $f(x) = 0$ has a root α in the interval $[3, 4]$ (x in radians).

$f(x)$ is continuous on the interval

$f(3) = -1.02 < 0$

$f(4) = 5.31 > 0$

A change of sign, so a root lies in $[3, 4]$

Answer: Sign change: $f(3) = -1.02$, $f(4) = 5.31$

3 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that no cube number can be of the form $9k + 5$, where k is an integer.

Cubes depend only on n mod 9: $n = 0, 1, 2, 3, 4, 5, 6, 7, 8$.

Remainders of n^3 on dividing by 9: 0, 1, 8, 0, 1, 8, 0, 1, 8

The only possible remainders are 0, 1, 8.

The remainder 5 never occurs.

Answer: remainder 5 never occurs among cubes (mod 9)

4 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(21, 0.3)$.

Find the smallest value of c such that $P(X \geq c) < 0.05$.

$P(X \geq 11) = 1 - P(X \leq 10) = 0.0264$

$P(X \geq 10) = 1 - P(X \leq 9) = 0.0676$

$0.0264 < 0.05$ but $0.0676 > 0.05$

Answer: $c = 11$

5 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 1$, $x \geq 0$.

Find $f^{-1}(x)$ and state its domain.

$f(x) = x^2 - 1$

Let $y = f(x)$: $x^2 = y + 1$

$x \geq 0$, so take the positive root: $x = \sqrt{y + 1}$

Answer: $f^{-1}(x) = \sqrt{x + 1}$, $x \geq -1$

6 Normal approximation to the Poisson (CIE)

$X \sim Po(40)$. Use a suitable approximation to find $P(X \geq 37)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(40, 40)$ approximately

Continuity correction: $P(Y > 36.5)$

$z = -0.553$, so probability = 0.710

Answer: 0.710

7 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$5\sin 2\theta + \cos\theta = 1.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 2\theta \approx 2\theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

$5 \times 2\theta + 1(1 - \frac{1}{2}\theta^2) = 1.5$

Multiply by 2 and rearrange: $\theta^2 - 20\theta + 1 = 0$

$\theta = \frac{20 \pm \sqrt{396}}{2}$, so $\theta = 0.0501$

Answer: a = 1, b = −20, c = 1; $\theta \approx 0.0501$

8 Passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is projected vertically upwards from the ground. It passes a ledge 3 s after projection and again 8 s after projection. Find (a) the speed of projection, (b) the height of the ledge, (c) the greatest height. Take g = 9.8.

Same height at t = 3 and t = 8, so the stone is at its highest point at t = 5.5
(a) At the top v = 0: $0 = u - 9.8\times5.5$, so u = 53.9
(b) $s = ut - 4.9t^2$ with t = 3: $s = 53.9\times3 - 4.9\times3^2 = 117.6$
(c) $v^2 = u^2 - 2gs$: $0 = 53.9^2 - 19.6s$, s = 148.225

Answer: (a) 53.9 m s^{-1} (b) 117.6 m (c) 148.225 m

9 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of
$y = \frac{x^2 + 12}{x + 2}$.

$\frac{dy}{dx} = \frac{(x + 2) \times 2x - (x^2 + 12) \times 1}{(x + 2)^2} = \frac{x^2 + 4x - 12}{(x + 2)^2}$

$\frac{dy}{dx} = 0$: $x^2 + 4x - 12 = 0$, so $(x - 2)(x + 6) = 0$

x = 2: y = 4; x = −6: y = −12

Answer: (−6, −12), (2, 4)

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is −16 and the coefficient of x^2 is 112. Find n and k.

$nk = -16$ and $\frac{n(n - 1)}{2}k^2 = 112$

$(n - 1)k = \frac{2 \times 112}{-16} = -14$ (dividing the second equation by the first)

$n\,k - (n - 1)\,k = k = -16 - (-14) = -2$

$n = \frac{-16}{-2} = 8$

Answer: n = 8, k = −2