Day 92 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 12 kg is pulled at constant speed along rough horizontal ground, $\mu = 0.5$, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{5}{12}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

Vertical: $R = 12g - P\sin\theta$

Horizontal: $P\cos\theta = \mu R = 0.5(12g - P\sin\theta)$

$P(\frac{12}{13} + \frac{5}{26}) = 0.5 \times 12g$

$P = 52.7$, $R = 97.3$

Answer: P = 52.7 N, R = 97.3 N

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 144 people: 48 Sales, 72 Admin and 24 Production.
A stratified sample of size 18 is taken.
Find the number of people sampled from each group.

Sampling fraction = $\frac{18}{144} = \frac{1}{8}$

Sales: $48 \times \frac{1}{8} = 6$

Admin: $72 \times \frac{1}{8} = 9$

Production: $24 \times \frac{1}{8} = 3$

Answer: Sales 6, Admin 9, Production 3

3 Change of sign to locate a root (All boards)

$f(x) = x^2 - 4\cos x - 12$.
Show that the equation f(x) = 0 has a root α in the interval [2, 3] (x in radians).

f(x) is continuous on the interval
$f(2) = -6.34 < 0$
$f(3) = 0.960 > 0$
A change of sign, so a root lies in [2, 3]
Answer: Sign change: f(2) = −6.34, f(3) = 0.960

4 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(2, 0) and B(−6, −6) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−2, 2) lies on the circle.

Centre = midpoint of AB = (−2, −3)
$r^2 = (4)^2 + (3)^2 = 25$
$(x + 2)^2 + (y + 3)^2 = 25$: $x^2 + y^2 + 4x + 6y - 12 = 0$
(b) $CM^2 = (0)^2 + (5)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 + 4x + 6y - 12 = 0$

5 Poisson over different intervals (CIE)

Faults are reported at random at an average of 6 per hour.
Find the probability that in a 90-minute period there are (a) zero events (b) at least 3.
(c) Find the probability that at least one occurs in each of two separate 90-minute periods.

$\lambda = 6 \times \frac{90}{60} = 9$, $X \sim Po(9)$

(a) $P(X = 0) = e^{-9} = 0.000123$
(b) $P(X \geq 3) = 1 - P(X \leq 2) = 0.994$
(c) $(1 - 0.000123)^2 = 1.00$
Answer: (a) 0.000123 (b) 0.994 (c) 1.00

6 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 11 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{55}{13}, \frac{132}{13})$. At time T the velocity is $(\frac{55}{13}, \frac{132}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{55}{13})^2 + \frac{132}{13}(\frac{132}{13} - 9.8T) = 0$

$11^2 - 9.8T \times \frac{132}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 11 \div (9.8 \times \frac{12}{13}) = \frac{715}{588}$

(b) $h = \frac{132}{13} \times \frac{715}{588} - 4.9 \times \frac{715}{588}^2 = \frac{10285}{2016}$

Answer: (a) T = 1.22 s (b) 5.10 m

7 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 4$.
Find the gradients of the two possible tangents and the points of contact.

Tangent at x = a: $y - (a^2 + 4) = 2a(x - a)$. Through (0, 0): $-a^2 - 4 = -2a^2$
So $a^2 = 4$ and $a = \pm 2$
Gradient $2a = \pm 4$
Answer: gradients ±4 at points (±2, 8)

8 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 3u_n - 3$.

Given that $u_3 = 42$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 3k - 3$, $u_3 = 3(3k - 3) - 3$
$9k - 12 = 42$ so $k = 6$
Terms: 6, 15, 42, 123
Sum = 186
Answer: (a) k = 6 (b) 186

9 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -6 \end{pmatrix}$ and $\begin{pmatrix} k \\ 5 \\ -1 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 5k + 6 = 0$
$(k + 2)(k + 3) = 0$

$k = -3$ or $k = -2$. For $k = -2$ the first vector is $\begin{pmatrix} -2 \\ -2 \\ -6 \end{pmatrix}$, with length $\sqrt{44} = 2\sqrt{11}$

Answer: k = −3 or −2; length $2\sqrt{11}$

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is -20 and the coefficient of x^2 is 180. Find n and k.

$nk = -20$ and $\frac{n(n-1)}{2}k^2 = 180$

$(n-1)k = \frac{2 \times 180}{-20} = -18$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = -20 - (-18) = -2$

$n = \frac{-20}{-2} = 10$

Answer: n = 10, k = −2