Day 91 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 4th term of an arithmetic sequence is 17 and the 10th term is 11.
Find the first term and common difference, and the sum of the first 12 terms.

$u_{10} - u_4 = 6\,d = 11 - 17$ so $d = -1$

$a + 3d = 17$ so $a = 20$

$S_{12} = \frac{12}{2}(2 \times 20 + 11 \times (-1))$

Answer: $a = 20$, $d = -1$, $S_{12} = 174$

2 Exponential growth and decay modelling (All boards)

A car costs £12000 new. Its value falls by 20% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £4000.

$V = 12000 \times 0.8^t$

$12000 \times 0.8^t = 4000$ gives $0.8^t = \frac{1}{3}$

$t = \frac{\ln(1/3)}{\ln 0.8} = 4.92$ years

Answer: (a) $V = 12000 \times 0.8^t$ (b) $t = 4.92$ years

3 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (2, −4).
Find the coordinates of the corresponding point on:

(a) $y = f(\frac{x}{2})$

(b) $y = f(x + 4)$

(c) $y = f(-x)$

(a) $y = f(\frac{x}{2})$: (4, −4)

(b) $y = f(x + 4)$: (−2, −4)

(c) $y = f(-x)$: (−2, −4)

A stretch or translation moves the point in the stated direction only.

Answer: (a) (4, −4); (b) (−2, −4); (c) (−2, −4)

4 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 39.2 m s^{-1} from a point h m above horizontal ground, and hits the ground 10 s later.
Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 39.2$, $a = -9.8$, $t = 10$

$-h = 39.2 \times 10 - 4.9 \times 10^2$, so $h = 98$

(b) At the highest point $v = 0$: $0 = 39.2^2 - 2 \times 9.8 \times s$

$s = 78.4$ m above the point of projection

Greatest height above the ground = 98 + 78.4

Answer: (a) 98 m (b) 176.4 m

5 Conditional probability and independence (All boards)

P(B) = 0.45, P(A) = 0.25 and P(A | B) = 0.45.
(a) Find P(A ∩ B) and P(A ∪ B).
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.45 \times 0.45 = \frac{81}{400}$

$P(A \cup B) = 0.25 + 0.45 - \frac{81}{400} = \frac{199}{400}$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{81}{400}}{0.25} = 0.81$

(c) $P(A)P(B) = 0.25 \times 0.45 = \frac{9}{80}$, which does not equal $P(A \cap B) = \frac{81}{400}$

So A and B are not independent

Answer: (a) $\frac{81}{400}$, $\frac{199}{400}$ (b) 0.81 (c) not independent

6 Integration by substitution (All boards)

Use the substitution u = cos x to find the exact value of

$\int_0^{\frac{\pi}{2}} \sin x \cos^3 x \, dx$.

$du = -\sin x \, dx$; x = 0 gives u = 1, $x = \frac{\pi}{2}$ gives u = 0

$I = \int_1^0 -u^3 \, du = \int_0^1 u^3 \, du = [\frac{u^4}{4}]_0^1$

$= \frac{1}{4}$

Answer: $\frac{1}{4}$

7 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 6x - 4y - 12 = 0$ passes through P(−6, 6).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−3, 2). Gradient of radius CP = $-\frac{4}{3}$

Gradient of tangent = $\frac{3}{4}$

$y - 6 = \frac{3}{4}(x + 6)$: $3x - 4y = -42$

y = 0: x = −14

Answer: (a) 3x − 4y = −42 (b) (−14, 0)

8 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 10, mean 48, sd 7;
group Y has n = 20, mean 66, sd 2.
Find the mean and standard deviation of all 30 values (3 s.f. for the sd).

$\Sigma x = 10 \times 48 + 20 \times 66 = 1800$, mean = 60

Σx^2 (A) $= 10(7^2 + 48^2) = 23530$

Σx^2 (B) $= 20(2^2 + 66^2) = 87200$

Variance = $\frac{110730}{30} - 60^2 = 91$, so sd = 9.54

Answer: mean 60, sd 9.54

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (5 m, 30 kg) rests with A on rough ground, $\mu = 0.2$, and B against a smooth wall, B being 3 m above the ground. A man of mass 75 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(5^2 - 3^2)} = 4$ m from the wall
Vertical: $R = (30 + 75)g$. Limiting friction: $F = \mu R = 0.2(30 + 75)g$
Horizontal: $S = F$ (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance $x \times 4/5$):
$S \times 3 = 30\,g \times 2 + 75\,g \times (4x \div 5)$
$21 \times 3 = 60 + 60x$ (g cancels), so $x = 0.05$
Answer: 0.05 m

10 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 10 cm. Angle AOB = $\frac{\pi}{2}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 25\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 50$

Segment area = sector $-$ triangle $= 25\pi - 50$

Chord AB = $2r\sin\frac{\theta}{2} = 10\sqrt{2}$; arc = $r\theta = 5\pi$

Perimeter = arc + chord $= 10\sqrt{2} + 5\pi$

Answer: area = $25\pi - 50$ cm^2, perimeter = $10\sqrt{2} + 5\pi$ cm