Day 90 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 84 m ahead. After a reaction time of 0.5 s she brakes with deceleration 4 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.5 v, braking distance = $v^2 \div (2\times4)$
$0.5\,v + v^2 \div 8 = 84$, so $v^2 + 4\,v - 672 = 0$
$(v - 24)(v + 28) = 0$, so $v = 24$
(b) Braking time = $24 \div 4 = 6$ s; total = 0.5 + 6
Answer: (a) 24 m s^{-1} (b) 6.5 s

2 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 - 1$, $y = t(t^2 - 16)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 16$, so $t = 0, 4, -4$
$t = 0$: $x = -1$
$t = \pm4$: $x = 31$
Answer: (−1, 0) and (31, 0)

3 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 2 m, mass 10 kg) has its centre of mass at G, AG = 0.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 1.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (10 + m)g$, so $T = \tfrac{1}{2}(10 + m)g$
Moments about A: $T\times2 = 10\,g\times0.25 + m\,g\times1.5$
$\tfrac{1}{2}(10 + m)\times2 = 2.5 + 1.5\,m \rightarrow 10 + 1\,m = 2.5 + 1.5\,m$
$m\,(1.5 - 1) = 10 - 2.5$, so $m = 15$
(b) $T = \tfrac{1}{2}(10 + 15)\times9.8$
Answer: (a) m = 15 (b) 122.5 N

4 Cubic inequalities (All boards)

Given that $(x - 4)$ is a factor of $f(x) = x^3 - 4x^2 - 4x + 16$, solve $f(x) > 0$.

Factorise: $f(x) = (x + 2)(x - 2)(x - 4)$
Roots at $x = -2, 2, 4$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) > 0$.
Answer: $-2 < x < 2$ or $x > 4$

5 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has $P(X = x) = k(7 - x)$ for $x = 1, 2, 3, 4$.

(a) Show that $k = \frac{1}{18}$. (b) Find E(X) and Var(X).

(c) Find $P(X \geq 3)$.

(a) $\Sigma\, P(X = x) = 1$: $6 + 5 + 4 + 3 = 18$, so $18k = 1$

(b) $E(X) = 1 \times \frac{1}{3} + 2 \times \frac{5}{18} + 3 \times \frac{2}{9} + 4 \times \frac{1}{6} = \frac{20}{9}$

$E(X^2) = \frac{55}{9}$, so $\mathrm{Var}(X) = \frac{55}{9} - (\frac{20}{9})^2 = \frac{95}{81}$

(c) $P(X \geq 3) = \frac{2}{9} + \frac{1}{6} = \frac{7}{18}$

Answer: (b) $E(X) = \frac{20}{9}$, $\mathrm{Var}(X) = \frac{95}{81}$ (c) $\frac{7}{18}$

6 Exponential equations reducing to quadratics (All boards)

Solve $2^{2x} - 182^x + 56 = 0$.
Give exact roots, or 3 s.f. where the answer is not exact.

Let $u = 2^x$: $u^2 - 18u + 56 = 0$, so $(u - 4)(u - 14) = 0$

$2^x = 4$ gives $x = 2$; $2^x = 14$ gives $x = \frac{\ln 14}{\ln 2} = 3.81$

Answer: $x = 2$ or $x = 3.81$

7 Modulus inequalities (All boards)

Solve $|x - 6| < |3x - 8|$.

Square both sides: $(x - 6)^2 < (3x - 8)^2$
$((x - 6) - (3x - 8))((x - 6) + (3x - 8)) < 0$
$(-2x + 2)(4x - 14) < 0$

Critical values $x = 1$ and $x = \frac{7}{2}$; sketch the (positive or negative) quadratic.

Answer: $x < 1$ or $x > \frac{7}{2}$

8 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 10 cm. Angle AOB = $\frac{\pi}{3}$.

Find the exact area of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = \frac{50\pi}{3}$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 25\sqrt{3}$

Segment area = sector − triangle = $\frac{50\pi - 75\sqrt{3}}{3}$

Answer: area = $\frac{50\pi - 75\sqrt{3}}{3}$ cm^2

9 Hypothesis test for a normal mean (All boards)

The mean mass of a sack (kg) is normally distributed with standard deviation 10; it was previously 33.
A random sample of 9 has mean 41.4.
Test, at the 5% level, whether the mean has changed.

$H_0: \mu = 33$, $H_1: \mu \neq 33$

$\bar{X} \sim N(33, \frac{10^2}{9})$, $z = \frac{41.4 - 33}{\frac{10}{3}} = 2.52$

p-value = 0.0117 (two-tailed) < 0.05
Reject H_0: evidence that the mean has changed.

Answer: $z = 2.52$, $p = 0.0117$; reject H_0

10 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^1 xe^{2x}\,dx$.

$u = x$, $\frac{dv}{dx} = e^{2x}$: $\int xe^{2x}\,dx = [\frac{xe^{2x}}{2}] - \int \frac{e^{2x}}{2}\,dx$

$= [\frac{xe^{2x}}{2} - \frac{e^{2x}}{4}]$

$= \frac{e^2 + 1}{4}$

Answer: $\frac{e^2 + 1}{4}$