Day 9 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 4 m s^{-1} and accelerates uniformly. She travels 105.6 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $105.6 = 4\times12 + \frac{1}{2}\times a\times12^2$, so $a = 0.8$
(b) $v = u + at = 4 + 0.8\times12 = 13.6$
(c) For the midpoint, $s = 52.8$: $v^2 = u^2 + 2as = 4^2 + 2\times0.8\times52.8$
$v^2 = 100.48$
Answer: (a) 0.8 m s^{-2} (b) 13.6 m s^{-1} (c) 10.0 m s^{-1}

2 Modulus equations (All boards)

Solve $|x^2 - 10x| = 24$.

$x^2 - 10x - 24 = 0$ gives $x = -2$ or $x = 12$
$x^2 - 10x + 24 = 0$ gives $x = 4$ or $x = 6$
Answer: $x = -2, 4, 6, 12$

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 25 pairs of values of age of a car and its price, $r = -0.46$.
The critical value for a two-tailed test at the 5% level is 0.3961.
Test, at this level, whether there is any correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
Critical value 0.3961
$|r| = 0.46 > 0.3961$
This is in the critical region, so reject H_0.
There is evidence of negative correlation.
Answer: Reject H_0: evidence of correlation

4 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 12 kg is pulled at constant speed along rough horizontal ground, $\mu = 0.3$, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{5}{12}$. Find P and the normal reaction. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$
Vertical: $R = 12g - P\sin\theta$
Horizontal: $P\cos\theta = \mu R = 0.3(12g - P\sin\theta)$
$P(\frac{12}{13} + \frac{3}{26}) = 0.3\times12g$
$P = 34.0$, $R = 105$
Answer: P = 34.0 N, R = 105 N

5 Binomial expansion: rational powers (All boards)

Expand $(1 - x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = \frac{3}{2}$, $y = -x$

Coefficients of 1, x, x^2: $1, -\frac{3}{2}, \frac{3}{8}$

Valid when $|x| < 1$, i.e. $|x| < 1$

Answer: $1 - \frac{3}{2}x + \frac{3}{8}x^2$; $|x| < 1$

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6 Quotient rule: stationary points (All boards)

Find the exact coordinates of the stationary point of

$y = \frac{\ln x}{x^2}$ for $x > 0$.

$$\frac{dy}{dx} = \frac{x^2 \times \frac{1}{x} - \ln x \times 2x}{(x^2)^2}$$

$$= \frac{1 - 2\ln x}{x^3}$$

$\frac{dy}{dx} = 0$: $\ln x = \frac{1}{2}$, so $x = e^{\frac{1}{2}}$

$y = \frac{1}{2} \div e = \frac{e^{-1}}{2}$

Answer: $\left(e^{\frac{1}{2}}, \frac{e^{-1}}{2}\right)$

7 Hypothesis test for a Poisson mean (CIE)

Accidents occur as a Poisson process at a mean rate of 8 per week.
After a change, the number recorded in one week is 5.
Test at the 10% level whether the mean rate has decreased.

$H_0: \lambda = 8$, $H_1: \lambda < 8$; $X \sim Po(8)$

$P(X \leq 5) = 0.191$

$0.191 > 0.1$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.191; do not reject H_0

8 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 40 cm^3s^{-1}. The surface radius is always $\frac{h}{4}$.

Find the rate at which the depth is rising when h = 2 cm.

$V = \frac{1}{3}\pi(h/4)^2h = \frac{\pi h^3}{48}$, so $\frac{dV}{dh} = \frac{\pi h^2}{16}$

$$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{40 \times 16}{\pi h^2}$$

h = 2: $\frac{dh}{dt} = \frac{160}{\pi}$ cm s^{-1}

Answer: $\frac{160}{\pi}$ cm/s

9 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_1^3 \frac{1}{x}\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = $\frac{1}{2}$; y-values at x = 1, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3: 1.0000, 0.6667, 0.5000, 0.4000, 0.3333

$T = \frac{1}{4}[1.0000 + 0.3333 + 2(0.6667 + 0.5000 + 0.4000)]$

T = 1.12

The curve is convex (y'' > 0) on this interval, so the chords lie above the curve: overestimate

Answer: 1.12, an overestimate

10 Geometric series: smallest n (All boards)

A geometric sequence has first term 200 and common ratio $\frac{4}{5}$.

Find the first term of the sequence that is less than 10.

$200 \times \frac{4}{5}^{n-1} < 10$

$\frac{4}{5}^{n-1} < \frac{1}{20}$

$(n - 1) \log \frac{4}{5} < \log \frac{1}{20}$; dividing by the negative log reverses the inequality:

$n - 1 > 13.43$, so $n = 15$

Check: $u_{14} = 10.995$, $u_{15} = 8.796$

Answer: n = 15 (the 15th term)