Day 89 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta + 6\tan\theta + 2 = 0$
for $0° \leq \theta \leq 360°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta + 6\tan\theta + 4 = 0$
$\tan\theta = -2$ or $\tan\theta = -1$
$\tan\theta = -2$: $\theta = 116.6°, 296.6°$
$\tan\theta = -1$: $\theta = 135.0°, 315.0°$
Answer: θ = 116.6°, 135.0°, 296.6°, 315.0°

2 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{24}{7}$, $\mu = 0.25$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8.

$\sin\alpha = 0.96$, $\cos\alpha = 0.28$, $R = 5g\cos\alpha = 13.72$ N
Limiting friction $= \mu R = 3.43$ N
P smallest: friction acts up the slope: $P + \mu R = 5g\sin\alpha$, $P = 47.04 - 3.43$
P greatest: friction acts down the slope: $P = 47.04 + 3.43$
Answer: 43.61 N ≤ P ≤ 50.47 N

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 12x + 10y + 25 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(1, −4) lies inside or outside the circle.

Complete the square: $(x + 6)^2 + (y + 5)^2 = 36$
Centre (−6, −5), radius 6
Distance^2 from centre to P $= 7^2 + 1^2 = 50$
Compare with $r^2 = 36$
Answer: (a) centre (−6, −5), radius 6 (b) P is outside the circle

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg carries a person of mass 40 kg. The cable tension is 5562 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 540 kg), taking upwards as positive: $T - 540g = 540a$
$5562 - 5292 = 540a$, so $a = 0.5$
(a) a is positive, so the acceleration is 0.5 m s^{-2} upwards
(b) Person: $R - 40g = 40a$, so $R = 40(9.8 + 0.5)$
Answer: (a) 0.5 m s^{-2} upwards (b) 412 N

5 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that

$$\frac{\cos 2\theta - \cos\theta}{\theta^2}$$

approaches as $\theta \to 0$.

$\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$

Numerator $\approx \frac{1-4}{2}\theta^2 = \frac{-3}{2}\theta^2$

Divide by θ^2: expression $\approx -\frac{3}{2}$

Answer: $-\frac{3}{2}$

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6 Normal approximation to the binomial (All boards)

X ~ B(120, 0.25). Use a normal approximation to estimate P(X ≤ 27) to 3 significant figures.

np = 30, npq = 22.5, so Y ~ N(30, 22.5)
With continuity correction: P(Y < 27.5)
$z = \frac{27.5 - 30}{4.74} = -0.527$, P(Z < −0.527) = 0.299
Answer: 0.299

7 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line 12x + 5y = k is a tangent to the circle
$(x + 3)^2 + (y + 1)^2 = 4$.
Find the two possible values of k.

Centre (−3, −1), radius 2
Distance from centre to the line = radius:
$\frac{|-41 - k|}{13} = 2$
|−41 − k| = 26
Answer: k = −67 or k = −15

8 Separable differential equations (All boards)

Solve the differential equation
$\frac{dy}{dx} = 2y \sin x$,
given that y = 1 when $x = \frac{\pi}{2}$. Give y in terms of x.

Separate: $\int \frac{1}{y}\, dy = \int 2\sin x\, dx$, so $\ln y = -2\cos x + C$
$x = \frac{\pi}{2}$, y = 1 gives 0 = 0 + C, so C = 0
$y = e^{-2\cos x}$
Answer: $y = e^{-2\cos x}$

9 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The life of a battery is $N(55, 8^2)$. A battery is classed as 'special' if its life is less than 40.
9 batterys are chosen at random. Find the probability that exactly 3 are special (3 s.f.).

P(special) = P(X < 40) = P(Z < −1.88) = 0.0304
Y ~ B(9, 0.0304)
$P(Y = 3) = C(9, 3)(0.0304)^3(0.970)^6 = 0.00196$
Answer: 0.00196

10 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{4} u_n + 15$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = \frac{1}{4} L + 15$ gives $L = \frac{15}{1 - \frac{1}{4}} = 20$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = (-20) \times \left(\frac{1}{4}\right)^{n-1}$

$|-20| \times \left(\frac{1}{4}\right)^{n-1} < \frac{1}{10}$

Check: n = 4 gives 0.31250, n = 5 gives 0.07812
Answer: (a) L = 20 (b) n = 5