Day 88 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 + 5$, $y = 2t(t^2 - 4)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 4$, so $t = 0, 2, -2$
$t = 0$: $x = 5$
$t = \pm 2$: $x = 17$
Answer: (5, 0) and (17, 0)

2 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^2 \sqrt{x^3 + 1}\, dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

$h = \frac{1}{2}$; y-values at $x = 0, \frac{1}{2}, 1, \frac{3}{2}, 2$: 1.0000, 1.0607, 1.4142, 2.0917, 3.0000

$T = \frac{1}{4}[1.0000 + 3.0000 + 2(1.0607 + 1.4142 + 2.0917)]$

$T = 3.28$
The curve is convex ($y'' > 0$) on this interval, so the chords lie above the curve: overestimate
Answer: 3.28, an overestimate

3 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$, and passes through A, 40 m horizontally from O and 10.4 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $40 = U\cos\alpha \times t$, so $t = 40 \div (0.8\,U)$
Vertical: $10.4 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$
$10.4 = 30 - 4.9\times 40^2 \div (0.64\,U^2)$
$U^2 = 12250 \div 19.6 = 625$, $U = 25$
Answer: 25 m s^{-1}

4 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a cube of edge x cm is increasing at 48 cm^{3}s^{-1}.
Find the rate of increase of its total surface area when $x = 4$.

$V = x^3$, $\frac{dV}{dx} = 3x^2$, so $\frac{dx}{dt} = \frac{48}{3x^2} = 1$ at $x = 4$

$S = 6x^2$, $\frac{dS}{dx} = 12x$, so $\frac{dS}{dt} = 12 \times 4 \times 1 = 48$

Answer: 48 cm^2/s

5 Modulus graph and a line (All boards)

The diagram shows the graphs of $y = |3x - 3|$ and $y = -x + 3$.
The graphs meet at A and B.
Find the coordinates of A and B.

$|3x - 3| = -x + 3$
Solve $3x - 3 = \pm(-x + 3)$.
$x = 0$ and $x = \frac{3}{2}$ (both checked in the original equation)
$y = 3$ and $y = \frac{3}{2}$
Answer: A(0, 3), B($\frac{3}{2}$, $\frac{3}{2}$)

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6 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = -\sqrt{2}\sin\theta$ for $-180° \le \theta \le 180°$.

$\sin 2\theta \equiv 2\sin\theta\cos\theta$: $\sin\theta(2\cos\theta + \sqrt{2}) = 0$
$\sin\theta = 0$: $\theta = -180°, 0°, 180°$

$\cos\theta = \frac{-\sqrt{2}}{2}$: $\theta = -135°, 135°$

Do not divide by $\sin\theta$, as that would lose solutions
Answer: $\theta = -180°, -135°, 0°, 135°, 180°$

7 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 kg, 2 m) is smoothly hinged at A to a vertical wall, with a 2 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 2 = 4g \times 1 + 2g \times 2$
$T\sin\theta = (2 + 2)g = 39.2$, so (a) $T = 49.0$
Horizontal component of hinge force: $X = T\cos\theta = 29.4$
Vertical: $Y + T\sin\theta = (4 + 2)g$, so $Y = 4g \div 2 = 19.6$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(29.4^2 + 19.6^2)}$
Answer: (a) 49 N (b) 35.3 N

8 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses in grams of two groups: factory A has n = 27, mean 44, sd 9;
factory B has n = 9, mean 20, sd 9.
Find the mean and standard deviation of all 36 values (3 s.f. for the sd).

$\Sigma x = 27 \times 44 + 9 \times 20 = 1368$, mean = 38
Σx^2 (A) $= 27(9^2 + 44^2) = 54459$
Σx^2 (B) $= 9(9^2 + 20^2) = 4329$
Variance $= \frac{58788}{36} - 38^2 = 189$, so sd = 13.7
Answer: mean 38, sd 13.7

9 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \times 2^{2x} - 30 \times 2^x + 48 = 0$.

Let $y = 2^x$. Then $3y^2 - 30y + 48 = 0$
$3(y - 8)(y - 2) = 0$
$y = 8$ or $y = 2$
$2^x = 8$ gives $x = 3$; $2^x = 2$ gives $x = 1$
Answer: $x = 1$ or $x = 3$

10 Venn diagrams and conditional probability (All boards)

$P(A) = 0.45$, $P(B) = 0.5$ and $P(A \cap B) = 0.05$.
Find
(a) $P(A \cup B)$
(b) $P(B \mid A)$
(c) $P(A \mid B')$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.45 + 0.5 - 0.05 = 0.9$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.45} = \frac{1}{9}$
(c) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.45 - 0.05}{1 - 0.5} = 0.8$
Answer: (a) 0.9 (b) $\frac{1}{9}$ (c) 0.8