Day 87 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 57.6 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $57.6 = 6\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.3$
(b) $v = u + at = 6 + 0.3\times8 = 8.4$
(c) For the midpoint, $s = 28.8$: $v^2 = u^2 + 2as = 6^2 + 2\times0.3\times28.8$
$v^2 = 53.28$
Answer: (a) 0.3 m s^{-2} (b) 8.4 m s^{-1} (c) 7.30 m s^{-1}

2 Differential equations in context (All boards)

A drink at 100°C is left in a room at 20°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 20)$.

After 15 minutes the temperature is 60°C. Find k, then the time for it to reach 40°C, to 3 s.f.

$\int \frac{1}{\theta - 20}\, d\theta = \int -k\, dt$, so $\ln(\theta - 20) = -kt + C$

$t = 0$: $\theta - 20 = 80$, so $\theta - 20 = 80e^{-kt}$

$t = 15$: $40 = 80e^{-15k}$, so $k = \frac{\ln(2)}{15} = 0.0462$

$20 = 80e^{-kt}$: $t = \frac{\ln(4)}{k} = 30.0$ minutes

Answer: k = 0.0462, t = 30.0 minutes

3 Poisson over different intervals (CIE)

Faults are reported at random at an average of 8 per hour.
Find the probability that in a 120-minute period there are (a) zero events (b) at least 2.
(c) Find the probability that at least one occurs in each of two separate 120-minute periods.

$\lambda = 8 \times \frac{120}{60} = 16$, $X \sim Po(16)$

(a) $P(X = 0) = e^{-16} = 0.000000113$
(b) $P(X \geq 2) = 1 - P(X \leq 1) = 1.00$
(c) $(1 - 0.000000113)^2 = 1.00$
Answer: (a) 0.000000113 (b) 1.00 (c) 1.00

4 Modulus equations (All boards)

Solve $|3x + 2| = -4x - 7$.

Case 1: $3x + 2 = -4x - 7$ gives $x = -\frac{9}{7}$

Case 2: $3x + 2 = -(-4x - 7)$ gives $x = -5$

Check $x = -\frac{9}{7}$: right-hand side $= -\frac{13}{7} < 0$, so reject

Check $x = -5$: right-hand side = 13 (valid)

Answer: x = −5

5 Normal approximation to the binomial (All boards)

$X \sim B(80, 0.3)$. Use a normal approximation to estimate $P(X \leq 24)$ to 3 significant figures.

$np = 24$, $npq = 16.8$, so $Y \sim N(24, 16.8)$
With continuity correction: $P(Y < 24.5)$

$z = \frac{24.5 - 24}{4.10} = 0.122$, $P(Z < 0.122) = 0.549$

Answer: 0.549

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6 Exponential growth and decay modelling (All boards)

A car costs £12000 new. Its value falls by 10% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £4000.

$V = 12000 \times 0.9^t$

$12000 \times 0.9^t = 4000$ gives $0.9^t = \frac{1}{3}$

$t = \frac{\ln(1/3)}{\ln 0.9} = 10.4$ years

Answer: (a) $V = 12000 \times 0.9^t$ (b) $t = 10.4$ years

7 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 12 kg, length 4 m) rests horizontally on supports at C and D, where AC = 0.5 m and DB = 1.5 m. A 8 kg particle is placed at E, AE = x m, and the plank is about to tilt about D. Find x.

About to tilt about D, so the reaction at C is zero and E lies between D and B.
Moments about D: $8g \times (x - 2.5) = 12g \times (2.5 - 2)$
$8(x - 2.5) = 6$
$x = 2.5 + 0.75$
Answer: $x = 3.25$

8 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{2}u_n + 5$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{2}L + 5$ gives $L = \frac{5}{1 - \frac{1}{2}} = 10$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = (-10) \times \left(\frac{1}{2}\right)^{n-1}$

$|-10| \times \left(\frac{1}{2}\right)^{n-1} < \frac{1}{100}$

Check: n = 10 gives 0.01953, n = 11 gives 0.00977
Answer: (a) $L = 10$ (b) $n = 11$

9 Tangent conditions (All boards)

The line $y = k - 2x$ is a tangent to the curve $y = \frac{8}{x}$.

Find the possible values of k.

$\frac{8}{x} = k - 2x$

$8 = kx - 2x^2$, so $2x^2 - kx + 8 = 0$

Tangent: discriminant $k^2 - 4 \times 2 \times 8 = 0$

$k^2 = 64$

Answer: $k = 8$ or $k = -8$

10 Differentiating a^x (All boards)

$y = 5^x$.

The gradient of the curve at P is 25 ln 5. Find the x-coordinate of P.

$\frac{dy}{dx} = \ln 5 \times 5^x$

$\ln 5 \times 5^x = 25 \ln 5$, so $5^x = 25$

$x = 2$, so $x = 2$

Answer: $x = 2$