Day 86 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(14, 0.15). Find, to 3 significant figures:
(a) P(X = 3) (b) P(X ≥ 3) (c) P(X ≤ 2)

(a) $C(14, 3)(0.15)^3(0.85)^{11} = 0.206$
(b) $1 - P(X \leq 2) = 1 - 0.648 = 0.352$
(c) $P(X \leq 2) = 0.648$
Answer: (a) 0.206 (b) 0.352 (c) 0.648

2 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (2, 3).
Find the coordinates of the corresponding point on:
(a) y = −f(x)

(b) $y = f(\frac{x}{2})$

(c) y = 3f(x)

(a) y = −f(x): (2, −3)

(b) $y = f(\frac{x}{2})$: (4, 3)

(c) y = 3f(x): (2, 9)
A stretch or translation moves the point in the stated direction only.
Answer: (a) (2, −3); (b) (4, 3); (c) (2, 9)

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Perth: mean 10.5, standard deviation 3.2;
Jacksonville: mean 20, standard deviation 5.4.
Make two comparisons.

Means: Jacksonville has the higher mean (20 against 10.5), so on average the mean windspeed is greater at Jacksonville
Standard deviations: Jacksonville is larger at Jacksonville (5.4 against 3.2)
so the readings at Jacksonville are more variable and those at Perth are more consistent
Answer: Jacksonville higher on average; Jacksonville more variable

4 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t^2 - 1$, $y = t^3 - 3t$.
Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where $t = -\frac{1}{2}$, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{3t^2 - 3}{2t}$

$t = -\frac{1}{2}$: point $(-\frac{3}{4}, \frac{11}{8})$, gradient $\frac{9}{4}$

Normal gradient $= -1 \div \frac{9}{4} = -\frac{4}{9}$, so normal: 32x + 72y − 75 = 0

Answer: $\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$**; 32x + 72y − 75 = 0**

5 Range of a composite function (All boards)

$f(x) = x^2 - 8x + 18$, $x \in \mathbb{R}$, and $g(x) = \frac{2}{x-1}$, $x > 1$.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x-4)^2 + 2 \geq 2$
So $f(x) \geq 2 > 1$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{2}{f(x)-1}$, with $f(x) \geq 2$, so $f(x) - 1 \geq 1$

Largest value is $\frac{2}{1} = 2$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 2$ (b) $0 < gf(x) \leq 2$

6 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled along rough horizontal ground, $\mu = 0.2$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{5}{12}$. The tension is 35 N. Find (a) the normal reaction, (b) the acceleration of the box. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

(a) Vertical: $R + 35\sin\theta = 4g$, so $R = \frac{1673}{65}$

(b) Horizontal: $35\cos\theta - \mu R = 4a$
$\frac{420}{13} - 0.2\times\frac{1673}{65} = 4a$
$a = 6.79$

Answer: (a) 25.7 N (b) 6.79 m s^{-2}

7 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2 x - 6\cos x - 2 = 0$
for $0° \leq x \leq 180°$.

Use $\sin^2 x \equiv 1 - \cos^2 x$: $2\cos^2 x + 6\cos x = 0$
Solving the quadratic: $\cos x = 0$ or $\cos x = -3$
$\cos x = 0$: $x = 90°$
$\cos x = -3$ has no solutions ($|\cos x| \leq 1$)
Answer: $x = 90°$

8 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 42.4 m above the sea at 15 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 12, vertical component 9 (upwards)
(a) $-42.4 = 9t - 4.9t^2$, so $4.9t^2 - 9t - 42.4 = 0$
$t = [9 + \sqrt{912.04}] \div 9.8 = 4$ (taking the positive root)
(b) Distance = 12×4
(c) Vertical speed = $9 - 9.8\times4 = -30.2$; speed = $\sqrt{(12^2 + 30.2^2)}$
Answer: (a) 4 s (b) 48 m (c) 32.5 m s^{-1}

9 Exponential equations reducing to quadratics (All boards)

Solve $5^{2x} - 29^x + 100 = 0$.
Give exact roots, or 3 s.f. where the answer is not exact.

Let $u = 5^x$: $u^2 - 29u + 100 = 0$, so $(u-25)(u-4) = 0$
$5^x = 25$ gives $x = 2$; $5^x = 4$ gives $x = \frac{\ln 4}{\ln 5} = 0.861$
Answer: $x = 2$ or $x = 0.861$

10 Tangent conditions (All boards)

The line $y = kx + 1$ is a tangent to the curve $y = 3x^2 - 2x + 4$.
(a) Find the possible values of k.
(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $3x^2 - 2x + 4 = kx + 1$

$3x^2 + (-2 - k)x + 3 = 0$

Tangent: discriminant = 0: $(-2 - k)^2 = 36$

$-2 - k = \pm 6$ so $k = 4$ or -8

$k = 4$: $x = \frac{k - (-2)}{6} = 1$, $y = 5$

Answer: (a) $k = -8$ or $k = 4$ (b) (1, 5)