Day 85 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Polynomial division (All boards)

Find the quotient and the remainder when $6x^3 - x^2 - 24x - 20$ is divided by $(3x + 4)$.

Quotient (matching leading terms in turn): $2x^2 - 3x - 4$

$(3x + 4)(2x^2 - 3x - 4) = 6x^3 - x^2 - 24x - 16$

Remainder $= 6x^3 - x^2 - 24x - 20 - (6x^3 - x^2 - 24x - 16) = -4$

Answer: quotient $2x^2 - 3x - 4$, remainder -4

2 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 - 2xy + 2y^2 = 15$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{6x - 2y}{-2x + 4y}$$

Horizontal tangent: $6x - 2y = 0$, so $y = 3x$

Substitute: $15x^2 = 15$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, 3)$ and $(-1, -3)$

Answer: $(1, 3)$ and $(-1, -3)$

3 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take $g = 9.8$.

Moments about C: $R_D \times 4 = 40\,g \times 2 + 10\,g \times 6$

$R_D = 343$

Resolving vertically: $R_C + R_D = 50\,g = 490$

$R_C = 490 - 343$

Answer: $R_C = 147$ N, $R_D = 343$ N

4 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 - 4$, $y = 2t(t^2 - 16)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 16$, so $t = 0, 4, -4$

$t = 0$: $x = -4$

$t = \pm 4$: $x = 28$

Answer: $(-4, 0)$ and $(28, 0)$

5 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $\mathbf{F_1} = (32\mathbf{i} + 15\mathbf{j})$ N and $\mathbf{F_2}$. Its acceleration is $(4\mathbf{i} + 2\mathbf{j})$ m s^{-2}. Find (a) $\mathbf{F_2}$, (b) the angle between $\mathbf{F_2}$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F_1} + \mathbf{F_2} = 5(4\mathbf{i} + 2\mathbf{j}) = 20\mathbf{i} + 10\mathbf{j}$

(a) $\mathbf{F_2} = (20\mathbf{i} + 10\mathbf{j}) - (32\mathbf{i} + 15\mathbf{j}) = -12\mathbf{i} - 5\mathbf{j}$

(b) $|\mathbf{F_2}| = \sqrt{(-12^2 + -5^2)} = 13$

$\cos\theta = -12 \div 13$, so $\theta = 157.4°$

Answer: (a) $\mathbf{F_2} = (-12\mathbf{i} - 5\mathbf{j})$ N (b) 157.4°

6 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 4\theta}{\theta \sin 3\theta}$ approaches as $\theta \to 0$.

$1 - \cos 4\theta \approx \frac{1}{2}(4\theta)^2 = \frac{16}{2}\theta^2$

$\sin 3\theta \approx 3\theta$

Expression $\approx \frac{\frac{16}{2}\theta^2}{\theta \times 3\theta} = \frac{16}{6} = \frac{8}{3}$

Answer: $\frac{8}{3}$

7 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The marks of two groups: class A has n = 32, mean 32, sd 7;
class B has n = 16, mean 23, sd 2.
Find the mean and standard deviation of all 48 values (3 s.f. for the sd).

$\Sigma x = 32 \times 32 + 16 \times 23 = 1392$, mean = 29

Σx^2 (A) $= 32(7^2 + 32^2) = 34336$

Σx^2 (B) $= 16(2^2 + 23^2) = 8528$

Variance $= \frac{42864}{48} - 29^2 = 52$, so sd = 7.21

Answer: mean 29, sd 7.21

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 10$.

Assume $x^2 - y^2 = (x - y)(x + y) = 10$.
$x - y$ and $x + y$ are both odd or both even (they differ by 2y).
Both odd: the product is odd, but 10 is even.
Both even: the product is a multiple of 4, but 10 is not.
Contradiction either way.
Answer: no such integers exist

9 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations x = 4sec t, y = 2tan t.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form y = mx + c.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{2}{4\sin t}$

$t = \frac{\pi}{4}$: gradient $m = \frac{\sqrt{2}}{2}$, point $(4\sqrt{2}, 2)$

$c = y - mx = -2$

Answer: $\frac{dy}{dx} = \frac{2}{4\sin t}$; $y = \frac{\sqrt{2}}{2}x - 2$

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(22, p). H_0: p = 0.4 is tested against H_1: p > 0.4 using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when p = 0.5 (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 14$

(b) Type I error = P(reject H_0 | p = 0.4) = 0.0215

(c) Type II error = P(not in critical region | p = 0.5)
$P(X \leq 13)$ when p = 0.5: 0.857
Answer: (a) $X \geq 14$ (b) 0.0215 (c) 0.857