Day 84 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 384 items.
The 5th item chosen is number 55 and the 8th item chosen is number 91.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(91 - 55) \div (8 - 5) = 12$
First item = $55 - 4 \times 12 = 7$
Sample size = $384 \div 12 = 32$
Answer: Interval 12, first 7, size 32

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\sin^2\theta + 3\cos\theta - 3 = 0$
for $0° \leq \theta \leq 180°$.

Use $\sin^2\theta \equiv 1 - \cos^2\theta$: $2\cos^2\theta - 3\cos\theta + 1 = 0$

Solving the quadratic: $\cos\theta = 1$ or $\cos\theta = \frac{1}{2}$

$\cos\theta = 1$: $\theta = 0°$

$\cos\theta = \frac{1}{2}$: $\theta = 60°$

Answer: $\theta = 0°, 60°$

3 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t - 1$, $y = 4t^2$.
Find a Cartesian equation of the curve.

From x: $t = x + 1$
$y = 4(x + 1)^2$
Expand.
Answer: $y = 4x^2 + 8x + 4$

4 Fixed point iteration (All boards)

The equation $x^3 = 2x + 6$ has a root α. Use the iteration $x_{n+1} = \sqrt[3]{2x + 6}$ with $x_0 = 3$
to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt[3]{2 \times 3 + 6} = 2.2894$
$x_2 = 2.1952$, $x_3 = 2.1821$
Continuing, the values settle at 2.1800, so $\alpha = 2.18$
Answer: $x_1 = 2.2894$, $x_2 = 2.1952$, $x_3 = 2.1821$; $\alpha = 2.18$

5 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 3$, $y = -3$ so $a = 3$, $b = -3$

Through $(2, -4)$: $-4 = \frac{k}{-1} - 3$

$k = (-4 + 3)(-1) = 1$

$x = 0$: $y = -\frac{10}{3}$; $y = 0$: $x = \frac{10}{3}$

Answer: $k = 1$, $a = 3$, $b = -3$; $(0, -\frac{10}{3})$ and $(\frac{10}{3}, 0)$

6 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table, $\mu = 0.2$, joined by a light string over a smooth pulley to particle B (2 kg), 1.5 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $2g - T = 2a$; A: $T - 0.2 \times 3g = 3a$
Adding: $(2 - 0.2 \times 3)g = 5a$, so $a = 2.744$
Speed of B and A as B lands: $v^2 = 2 \times 2.744 \times 1.5 = 8.232$
After B lands, A decelerates at $\mu g = 0.2 \times 9.8 = 1.96$
$0 = v^2 - 2(\mu g)s$, $s = 8.232 \div 3.92$

Answer: (a) 2.744 m s^{-2} (b) 2.1 m

7 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with $P(X < 84) = 0.1587$ and $P(X > 96) = 0.1587$.
Find μ and σ.

z-values: 84: $z = -1.00$; 96: $z = 1.00$
$84 = \mu - 1.00\,\sigma$ and $96 = \mu + 1.00\,\sigma$
Subtract: $12 = 2.00\,\sigma$, so $\sigma = 6.00$
$\mu = 84 + 1.00 \times 6.00 = 90.0$

Answer: $\mu = 90$, $\sigma = 6$

8 Chain rule and gradients (All boards)

$y = (\ln x)^4$ for $x > 0$.
Find $\frac{dy}{dx}$ and its exact value at $x = e^2$.

Let $u = \ln x$: $y = u^4$, so $\frac{dy}{dx} = 4u^3 \times \frac{1}{x} = \frac{4(\ln x)^3}{x}$

At $x = e^2$: $\ln x = 2$, so gradient $= \frac{32}{e^2}$

Answer: $\frac{dy}{dx} = \frac{4(\ln x)^3}{x}$; gradient $= \frac{32}{e^2}$

9 Inverse of a quadratic function (All boards)

$f(x) = x^2 + 2x - 4$, $x \geq -1$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = (x + 1)^2 - 5$
Let $y = f(x)$: $(x + 1)^2 = y + 5$
$x \geq -1$, so take the positive root: $x = -1 + \sqrt{y + 5}$

Answer: $f^{-1}(x) = -1 + \sqrt{x + 5}$, $x \geq -5$

10 Projectile through a given point (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$, and passes through A, 5 m horizontally from O and 7.1 m above the ground. Find the speed of projection. Take $g = 9.8$.

Let the speed be U. Horizontal: $5 = U\cos\alpha \times t$, so $t = 5 \div (\frac{5}{13}\,U)$

Vertical: $7.1 = U\sin\alpha \times t - 4.9t^2$
Eliminate t: $y = x\tan\alpha - 4.9x^2 \div (U^2\cos^2\alpha)$

$7.1 = 12 - 4.9 \times 5^2 \div (\frac{25}{169}\,U^2)$

$U^2 = 828.1 \div 4.9 = 169$, $U = 13$

Answer: 13 m s^{-1}