Day 82 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Algebraic fractions (All boards)

Express $\frac{5}{(x-2)} - \frac{3}{(x-5)}$ as a single fraction in its simplest form.

Common denominator $(x-2)(x-5)$

Numerator: $5(x-5) - 3(x-2) = 2x - 19$

Answer: $\mathbf{\frac{2x-19}{(x-2)(x-5)}}$

2 Index equations solved with logarithms (All boards)

Solve $2^{2x-3} = 5^{2x-2}$.

Give your answer to 3 significant figures.

Take logs: $(2x-3)\ln 2 = (2x-2)\ln 5$

Collect x terms: $x(2\ln 2 - 2\ln 5) = -2\ln 5 + 3\ln 2$

$x = \frac{-2\ln 5 + 3\ln 2}{2\ln 2 - 2\ln 5}$

$x = 0.622$

Answer: x = 0.622

3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 720 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(100 + 60) × V

720 = 80 V, so V = 9

(b) Deceleration = V ÷ 15 = 9 ÷ 15

Answer: (a) V = 9 (b) 0.6 m s^{-2}

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 6 pairs of values of daily rainfall and ice cream sales, r = 0.87.

The critical value for a one-tailed test at the 5% level is 0.7293.

Test, at this level, whether there is positive correlation between the two variables.

H_0: $\rho = 0$, H_1: $\rho > 0$

Critical value 0.7293

r = 0.87 > 0.7293

This is in the critical region, so reject H_0.

There is evidence of positive correlation.

Answer: Reject H_0: evidence of correlation

5 Factor theorem and remainder theorem (All boards)

$f(x) = x^3 + ax^2 + bx + 36$, where a and b are constants.

$(x-3)$ is a factor of f(x). The remainder when f(x) is divided by $(x-1)$ is 24.

(a) Find a and b. (b) Solve f(x) = 0.

f(3) = 0: $27 + 9a + 3b + 36 = 0$

f(1) = 24: $1 + a + b + 36 = 24$

Solving simultaneously: $a = -4$, $b = -9$

$f(x) = x^3 - 4x^2 - 9x + 36 = (x-3)(x-4)(x+3)$

Answer: (a) a = −4, b = −9 (b) x = −3, 3, 4

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6 Poisson over different intervals (CIE)

Customers enter a shop at random at an average of 2 per hour.
Find the probability that in a 30-minute period there are (a) zero events (b) at least 3.
(c) Find the probability that at least one occurs in each of two separate 30-minute periods.

$\lambda = 2 \times \frac{30}{60} = 1$, $X \sim Po(1)$

(a) $P(X = 0) = e^{-1} = 0.368$
(b) $P(X \geq 3) = 1 - P(X \leq 2) = 0.0803$
(c) $(1 - 0.368)^2 = 0.400$
Answer: (a) 0.368 (b) 0.0803 (c) 0.400

7 Work-energy on a rough slope (CIE)

A 8 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.25$, and slides 2 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 2 m. Take $g = 10$.

$R = 8g\cos\alpha = 64$ N, $F = \mu R = 16$ N
(a) Work done against friction $= F \times d = 16 \times 2$
(b) Loss of PE $= 8g \times 2\sin\alpha = 96$ J
Work-energy: $\frac{1}{2} \times 8 \times v^2 = 96 - 32 = 64$
$v^2 = 16$
Answer: (a) 32 J (b) 4 m s^{-1}

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

lg y = lg A + n lg x: gradient n, intercept lg A

$n = \frac{7 - 5}{2 - 1} = 2$

At (1, 5): lg A $= 5 - 2 \times 1 = 3$, so $A = 10^3 = 1000$
Answer: n = 2, A = 1000

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{11}$ is irrational.

Assume $\sqrt{11} = \frac{a}{b}$ with a, b integers with no common factor.

Then $11b^2 = a^2$, so a^2 is a multiple of 11, hence a is a multiple of 11.
Write $a = 11c$. Then $11b^2 = 121c^2$, so $b^2 = 11c^2$.
So b is also a multiple of 11, contradicting no common factor.
Answer: contradiction, so $\sqrt{11}$ is irrational

10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $4x^2 + xy + y^2 = 60$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{8x + y}{x + 2y}$
Horizontal tangent: $8x + y = 0$, so $y = -8x$
Substitute: $60x^2 = 60$, so $x^2 = 1$ and $x = \pm 1$
Points: $(1, -8)$ and $(-1, 8)$
Answer: (1, −8) and (−1, 8)