Day 81 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Combined inequalities (All boards)

(a) Solve $5x + 3 > -3$.

(b) Solve $x^2 - x - 6 \geq 0$.

(c) Hence find the values of x satisfying both inequalities.

(a) $5x + 3 > -3$ so $x > -\frac{6}{5}$

(b) $(x + 2)(x - 3) \geq 0$: critical values -2 and 3

(b) gives $x \leq -2$ or $x \geq 3$

(c) Overlap of the two sets.

Answer: (a) $x > -\frac{6}{5}$ (b) $x \leq -2$ or $x \geq 3$ (c) $x \geq 3$

2 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has $P(X = x) = k(x^2 + 1)$ for $x = 0, 1, 2, 3$.

(a) Show that $k = \frac{1}{18}$. (b) Find E(X) and Var(X).

(c) Find $P(X \geq 2)$.

(a) $\Sigma P(X = x) = 1$: $1 + 2 + 5 + 10 = 18$, so $18k = 1$

(b) $E(X) = 1 \times \frac{1}{9} + 2 \times \frac{5}{18} + 3 \times \frac{5}{9} = \frac{7}{3}$

$E(X^2) = \frac{56}{9}$, so $Var(X) = \frac{56}{9} - (\frac{7}{3})^2 = \frac{7}{9}$

(c) $P(X \geq 2) = \frac{5}{18} + \frac{5}{9} = \frac{5}{6}$

Answer: (b) $E(X) = \frac{7}{3}$, $Var(X) = \frac{7}{9}$ (c) $\frac{5}{6}$

3 Line meets curve: length of chord (All boards)

The line $y = 2x + 3$ meets the curve $y = x^2 + 7x + 3$ at A and B.

(a) Find the coordinates of A and B.

(b) Find the exact length AB.

$x^2 + 7x + 3 = 2x + 3$ gives $x^2 + 5x = 0$

$(x + 5)x = 0$: $x = -5$ or 0

A$(-5, -7)$, B$(0, 3)$

$AB^2 = 5^2 + 10^2 = 125$

Answer: A$(-5, -7)$, B$(0, 3)$; AB = $5\sqrt{5}$

4 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -2 \end{pmatrix}$ and $\begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - 2k - 8 = 0$

$(k + 2)(k - 4) = 0$

$k = -2$ or $k = 4$. For $k = 4$ the first vector is $\begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$, with length $\sqrt{36} = 6$

Answer: k = -2 or 4; length 6

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5 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 20 m s^{-1} from a point 4.9 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $4.9 = \frac{1}{2} \times 9.8 \times t^2$, so t = 1
(a) Horizontal distance = 20 × 1
(b) Vertical speed on landing = 9.8×1 = 9.8; horizontal speed = 20
Speed = $\sqrt{(20^2 + 9.8^2)} = 22.3$
$\tan\theta = 9.8 \div 20$, $\theta = 26.1°$
Answer: (a) 20 m (b) 22.3 m s^{-1} at 26.1° below the horizontal

6 Equations with sec x and tan x (All boards)

Solve $4\sec^2\theta - 10\tan\theta + 2 = 0$
for $-180° \leq \theta \leq 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $4\tan^2\theta - 10\tan\theta + 6 = 0$
$\tan\theta = \frac{3}{2}$ or $\tan\theta = 1$
$\tan\theta = \frac{3}{2}$: $\theta = -123.7°, 56.3°$
$\tan\theta = 1$: $\theta = -135.0°, 45.0°$
Answer: $\theta = -135.0°, -123.7°, 45.0°, 56.3°$

7 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 9 pairs of (x, y): $\Sigma x = 151$, $\Sigma y = 455$, $\Sigma x^2 = 3005$, $\Sigma xy = 8340$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 8.

$S_{xx} = 3005 - \frac{151^2}{9} = 472$, $S_{xy} = 8340 - \frac{151 \times 455}{9} = 706$

$b = \frac{S_{xy}}{S_{xx}} = 1.50$
$a = \bar{y} - b\,\bar{x} = 50.6 - (1.50)(16.8) = 25.4$
(b) $y = 25.4 + (1.50)(8) = 37.4$
Answer: (a) y = 25.4 + 1.50x (b) 37.4

8 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{3}{2}$ and $u_{n+1} = \frac{1}{1 - u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{50} and $\sum_{r=1}^{50} u_r$.

$u_2 = -2$, $u_3 = \frac{1}{3}$, $u_4 = \frac{3}{2}$
$u_4 = u_1$, so the sequence repeats with period 3.
$50 = 3 \times 16 + 2$, so $u_{50} = u_2 = -2$

Sum of one period = $-\frac{1}{6}$; 16 full periods plus 2 extra term(s)

Answer: (a) $-2, \frac{1}{3}$ (b) $u_{50} = -2$, sum = $-\frac{19}{6}$

9 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a cube of edge x cm is increasing at 36 cm^3s^{-1}.
Find the rate of increase of its total surface area when x = 2.

$V = x^3$, $\frac{dV}{dx} = 3x^2$, so $\frac{dx}{dt} = \frac{36}{3x^2} = 3$ at x = 2

$S = 6x^2$, $\frac{dS}{dx} = 12x$, so $\frac{dS}{dt} = 12 \times 2 \times 3 = 72$

Answer: 72 cm^2/s

10 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 11 m s^{-1}. 11 s later car B leaves P from rest, in the same direction, with acceleration 1.5 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 11) seconds.
B: ½×1.5×s^2 = 0.75 s^2 A: 11(s + 11)
0.75 s^2 − 11 s − 121 = 0 so s = 22 (s > 0)
(b) PQ = 0.75×22^2 = 363
(c) v = 0 + at = 1.5×22 = 33

Answer: (a) 22 s (b) 363 m (c) 33 m s^{-1}