Day 80 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Power, driving force and resistance (CIE)

A 1200 kg car has engine power 60 kW and constant resistance 500 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 24 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10. Give answers to 3 s.f. if not exact.

(a) Steady speed: driving force = resistance. P = Fv: 60000 = 500 v
(b) Driving force F = P ÷ v = 60000 ÷ 24 = 2500 N
Up the hill: F − 500 − 1200 g sin α = 1200 a
2500 − 500 − 600 = 1200 a
$a = \frac{7}{6}$

Answer: (a) 120 m s^{-1} (b) 1.17 m s^{-2}

2 Inverse of a rational function (All boards)

$f(x) = \frac{2x-2}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{2x-2}{2x-1}$: $y(2x-1) = 2x-2$
$2xy - y = 2x - 2$
$x(2y-2) = y - 2$
$x = \frac{y-2}{2y-2}$, then swap x and y

Answer: $f^{-1}(x) = \frac{x-2}{2x-2}$, $x \neq 1$

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean temperature (°C) in the same month are:
Perth: mean 27.5, standard deviation 5.4;
Jacksonville: mean 10.5, standard deviation 3.4.
Make two comparisons.

Means: Perth has the higher mean (27.5 against 10.5), so on average the mean temperature is greater at Perth
Standard deviations: Perth is larger at Perth (5.4 against 3.4)
so the readings at Perth are more variable and those at Jacksonville are more consistent
Answer: Perth higher on average; Perth more variable

4 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve C: y = x(4 − x) and the line L: y = 1x.
Find the area of the shaded region between C and L.

Intersect: x(4 − x) = 1x gives x = 0 or x = 3
Area = $\int(C - L)\,dx = \int_0^3 (3x - x^2)\,dx = [\frac{3x^2}{2} - \frac{x^3}{3}]$
$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$

Answer: $\frac{9}{2}$

5 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = −1
Line AB: y − 4 = 0(x − −3). Solve to get D = (−1, 4)
Area = ½ × AB × CD = 16
Answer: (a) x = −1 (b) D(−1, 4), area 16

6 Work-energy on a rough slope (CIE)

A 6 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.5$, and slides 4 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 4 m. Take $g = 10$.

$R = 6g\cos\alpha = 48$ N, $F = \mu R = 24$ N
(a) Work done against friction $= F \times d = 24 \times 4$
(b) Loss of PE $= 6g \times 4\sin\alpha = 144$ J
Work-energy: $\frac{1}{2}\times 6 \times v^2 = 144 - 96 = 48$
$v^2 = 16$
Answer: (a) 96 J (b) 4 m s^{-1}

7 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x + 4y = 25$ meets the circle $(x + 1)^2 + (y - 3)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre $(-1, 3)$, radius 6. Distance from centre to line:

$$d = \frac{|-16|}{5} = \frac{16}{5}$$

$$\text{Half-chord}^2 = 36 - \left(\frac{16}{5}\right)^2 = \frac{644}{25}$$

$$AB = 2\sqrt{\frac{644}{25}}$$

Answer: $\mathbf{\frac{20\sqrt{161}}{25}}$

8 Separable differential equations (All boards)

Solve the differential equation

$\frac{dy}{dx} = 4xy^2$,

given that $y = 1$ when $x = 0$. Give y in terms of x.

Separate: $\int \frac{1}{y^2}\,dy = \int 4x\,dx$, so $-\frac{1}{y} = 2x^2 + C$

$x = 0$, $y = 1$ gives $C = -\frac{1}{1}$

$$-\frac{1}{y} = 2x^2 - \frac{1}{1}$$

Answer: $\mathbf{y = \frac{1}{1 - 2x^2}}$

9 Normal approximation to the Poisson (CIE)

$X \sim \text{Po}(18)$. Use a suitable approximation to find $P(X \leq 24)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(18, 18)$ approximately
Continuity correction: $P(Y < 24.5)$
$z = 1.53$, so probability = 0.937
Answer: 0.937

10 Arc length and sector area (All boards)

A sector of a circle has area 14.4 cm^2 and angle 1.8 radians.
Find the radius, the arc length and the perimeter of the sector.

Area $= \frac{1}{2}r^2\theta$: $14.4 = \frac{1}{2}r^2 \times 1.8$, so $r^2 = 16 = 4^2$

$r = 4$ cm; arc length $s = r\theta = 4 \times 1.8 = 7.2$ cm
Perimeter $= 2r + s = 8 + 7.2 = 15.2$ cm
Answer: r = 4 cm, arc 7.2 cm, perimeter 15.2 cm