Day 8 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{4}{x+4} + 2$, is translated by $\begin{pmatrix}2\\3\end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = −4, y = 2

Translate by $\begin{pmatrix}2\\3\end{pmatrix}$: x = −2, y = 5

New curve: y = $\frac{4}{x+2} + 5$

y = 0: $\frac{4}{x+2} = -5$, so $x + 2 = -\frac{4}{5}$

Answer: x = −2, y = 5; crosses x-axis at $(-\frac{14}{5}, 0)$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A club has 288 members: 60 Bronze, 96 Silver and 132 Gold.

A stratified sample of size 24 is taken.

Find the number of members sampled from each group.

Sampling fraction = $\frac{24}{288} = \frac{1}{12}$

Bronze: $60 \times \frac{1}{12} = 5$

Silver: $96 \times \frac{1}{12} = 8$

Gold: $132 \times \frac{1}{12} = 11$

Answer: Bronze 5, Silver 8, Gold 11

3 Exponential equations reducing to quadratics (All boards)

Solve $4^x + 2^x - 20 = 0$.

Give your answer to 3 significant figures if not exact.

$4^x = (2^x)^2$. Let $u = 2^x$: $u^2 + 1u - 20 = 0$

$(u - 4)(u + 5) = 0$, so u = 4 or u = −5

2^x is always positive, so reject u = −5; $2^x = 4$ gives x = 2

Answer: x = 2

4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $9^{x-2} = 27^{x-1}$.

Write both sides as powers of 3.

$(3^2)^{x-2} = (3^3)^{x-1}$

$2(x - 2) = 3(x - 1)$

$2x - 4 = 3x - 3$

$-x = 1$

Answer: x = −1

5 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 30 $cm^3 s^{-1}$. The surface radius is always $\frac{h}{3}$.

Find the rate at which the depth is rising when h = 2 cm.

$V = \frac{1}{3}\pi(h/3)^2 h = \frac{\pi h^3}{27}$, so $\frac{dV}{dh} = \frac{\pi h^2}{9}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{30 \times 9}{\pi h^2}$

h = 2: $\frac{dh}{dt} = \frac{135}{2\pi}$ cm s^{-1}

Answer: $\frac{135}{2\pi}$ cm/s

6 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 29.4 m above the ground. At the same instant ball B is projected upwards at 14.7 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$
Height of B above the ground: $14.7\,t - 4.9t^2$
They meet when the distances add up to 29.4: $4.9t^2 + (14.7\,t - 4.9t^2) = 29.4$
so $14.7\,t = 29.4$, $t = 2$
Height of B = $14.7\times2 - 4.9\times2^2 = 9.8$
Answer: (a) 2 s (b) 9.8 m

7 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−8, −9) and B(4, 7) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(−10, −7) lies on the circle.

Centre = midpoint of AB = (−2, −1)
$r^2 = (-6)^2 + (-8)^2 = 100$
$(x + 2)^2 + (y + 1)^2 = 100$: $x^2 + y^2 + 4x + 2y - 95 = 0$
(b) $CM^2 = (-8)^2 + (-6)^2 = 100 = r^2$
Answer: (a) $x^2 + y^2 + 4x + 2y - 95 = 0$

8 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(23, p). H_0: p = 0.2 is tested against H_1: p ≠ 0.2 at the 5% level.
(a) Find the critical region.
(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. P(X ≤ 0) = 0.00590, P(X ≤ 1) = 0.0398
P(X ≥ 10) = 0.00894, P(X ≥ 9) = 0.0273
Critical region: X ≤ 0 or X ≥ 10
(b) Actual level = 0.00590 + 0.00894 = 0.0148
Answer: (a) X ≤ 0 or X ≥ 10 (b) 0.0148

9 Implicit differentiation: products and cubes (All boards)

A curve has equation $3x^2y - 2y^3 - 4x = 2$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (0, −1).

Differentiate: $6xy + 3x^2\frac{dy}{dx} - 6y^2\frac{dy}{dx} - 4 = 0$
$\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 - 6y^2}$
At (0, −1): $\frac{dy}{dx} = -\frac{2}{3}$, so the normal has gradient $\frac{3}{2}$
Answer: $\frac{dy}{dx} = -\frac{6xy - 4}{3x^2 - 6y^2}$; normal gradient $\frac{3}{2}$

10 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (12 kg, 2 m) is smoothly hinged at A to a vertical wall, with a 5 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{4}{3}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 2 = 12\,g \times 1 + 5\,g \times 2$
$T\sin\theta = (6 + 5)g = 107.8$, so (a) T = 135
Horizontal component of hinge force: $X = T\cos\theta = 80.9$
Vertical: $Y + T\sin\theta = (12 + 5)g$, so $Y = 12\,g \div 2 = 58.8$
(b) Force = $\sqrt{(X^2 + Y^2)} = \sqrt{(80.9^2 + 58.8^2)}$
Answer: (a) 135 N (b) 100 N