Day 79 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 4th term of an arithmetic sequence is 5 and the 11th term is 12.
Find the first term and common difference, and the sum of the first 15 terms.

$u_{11} - u_4 = 7\ d = 12 - 5$ so $d = 1$

$a + 3d = 5$ so $a = 2$

$S_{15} = \frac{15}{2}(2 \times 2 + 14 \times 1)$

Answer: a = 2, d = 1, $S_{15} = 135$

2 Differentiating a^x (All boards)

$y = 3^x$.
Find the exact gradient of the curve at $x = 2$ and the equation of the tangent there.

$\frac{dy}{dx} = 1 \times 1 \ln 3 \times 3^x = \ln 3 \times 3^x$

At $x = 2$: $y = 9$ and gradient $= 9 \ln 3$

Tangent: $y = (9 \ln 3)x + 9 - 18 \ln 3$

Answer: gradient $9 \ln 3$; $y = (9 \ln 3)x + 9 - 18 \ln 3$

3 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 4 men and 7 women. A team of 3 is chosen at random.
Find the probability that the team contains at least one of the women.

Total teams = C(11, 3) = 165

Teams of all men = C(4, 3) = 4

P(at least one wome) = $1 - \frac{4}{165}$

$= \frac{161}{165}$

Answer: $\frac{161}{165}$

4 Quadratic from its graph (All boards)

The diagram shows the curve with equation $y = f(x)$, where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point $(1, -1)$: $f(x) = a(x - 1)^2 - 1$

$f(0) = 1$: $a(1)^2 - 1 = 1$

$a = 2$, then expand

Answer: $f(x) = 2x^2 - 4x + 1$

5 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 6 m, mass 4 kg) has its centre of mass at G, AG = 0.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 4.25 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (4 + m)g$, so $T = \frac{1}{2}(4 + m)g$

Moments about A: $T \times 6 = 4g \times 0.25 + mg \times 4.25$

$\frac{1}{2}(4 + m) \times 6 = 1 + 4.25m \rightarrow 12 + 3m = 1 + 4.25m$

$m(4.25 - 3) = 12 - 1$, so $m = 8.8$

(b) $T = \frac{1}{2}(4 + 8.8) \times 9.8$

Answer: (a) m = 8.8 (b) 62.72 N

6 R-form and solving a sin x + b cos x = c (All boards)

(a) Express 6 sin x + 8 cos x in the form R sin(x + α), where R > 0 and 0° < α < 90°. Give α to 1 d.p.
(b) Solve 6 sin x + 8 cos x = 3 for 0° ≤ x < 360°, giving answers to 1 d.p.

$R^2 = 6^2 + 8^2 = 100$, so R = 10; $\tan\alpha = \frac{8}{6}$ gives α = 53.1°

$10\sin(x + 53.1°) = 3$, so $\sin(x + 53.1°) = \frac{3}{10}$

Principal value 17.5°, then use the symmetry of the sine graph
x = 109.4°, 324.3°
Answer: (a) R = 10, α = 53.1° (b) x = 109.4°, 324.3°

7 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 15 and the coefficient of x^2 is 90. Find n and k.

$nk = 15$ and $\frac{n(n-1)}{2}k^2 = 90$

$(n-1)k = \frac{2 \times 90}{15} = 12$ (dividing the second equation by the first)

$nk - (n-1)k = k = 15 - 12 = 3$

$n = \frac{15}{3} = 5$

Answer: n = 5, k = 3

8 Interpolation for median and percentiles (All boards)

The table summarises the length, l mm, of leaves of 49 items.
Use linear interpolation to estimate the upper quartile.

Position = 0.75 × 49 = 36.75
Cumulative frequency before the class 20 ≤ l < 30 is 24

$\text{Estimate} = 20 + \frac{36.75 - 24}{16} \times 10$

= 28.0
Answer: 28.0

9 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (9 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 0.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: 1 g − T = 1 a
A: T = 9 a
Adding: 1 g = 10 a, so a = 0.98
(a) T = 9 × 0.98 = 8.82
(b) $v^2 = 0 + 2 \times 0.98 \times 0.5 = 0.98$
Answer: (a) 8.82 N (b) 0.990 m s^{-1}

10 Double and triple angle values (All boards)

θ is acute and tan θ = 5.

Show that $\cos 2\theta = -\frac{12}{13}$ and find the exact value of sin 2θ.

$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - 5^2}{1 + 5^2} = -\frac{12}{13}$

$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2 \times 5}{1 + 5^2} = \frac{5}{13}$

Answer: $\cos 2\theta = -\frac{12}{13}$, $\sin 2\theta = \frac{5}{13}$