**Day 78** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (10 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 4 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle $\theta$ to it, $\tan\theta = \frac{5}{12}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T \sin\theta \times 4 = 10\,g \times 2 + 4\,g \times 4$
$T \sin\theta = (5 + 4)g = 88.2$, so (a) $T = 229$
Horizontal component of hinge force: $X = T\cos\theta = 212$
Vertical: $Y + T\sin\theta = (10 + 4)g$, so $Y = 10\,g \div 2 = 49$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(212^2 + 49^2)}$
**Answer: (a) 229 N (b) 217 N**

## 2 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small x (in radians), use the small angle approximations to find the value that $\frac{1 - \cos 5x}{x \sin 4x}$ approaches as $x \to 0$.

$1 - \cos 5x \approx \frac{1}{2}(5x)^2 = \frac{25}{2}x^2$

$\sin 4x \approx 4x$

Expression $\approx \frac{\frac{25}{2}x^2}{x \times 4x} = \frac{25}{8} = \frac{25}{8}$

**Answer: $\frac{25}{8}$**

## 3 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle rests on a rough plane inclined at $\alpha$, $\tan\alpha = \frac{7}{24}$, $\mu = 0.2$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.28$, $\cos\alpha = 0.96$, $R = 5\,g\cos\alpha = 47.04$ N
Limiting friction $= \mu R = 9.408$ N
P smallest: friction acts up the slope: $P + \mu R = 5\,g\sin\alpha$, $P = 13.72 - 9.408$
P greatest: friction acts down the slope: $P = 13.72 + 9.408$
**Answer: 4.312 N ≤ P ≤ 23.1 N**

## 4 Product rule: stationary points (All boards)

$y = (6x - 3)e^{-2x}$.
Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 6\,e^{-2x} - 2(6x - 3)e^{-2x} = (-12x + 12)e^{-2x}$

$e^{-2x} > 0$, so $\frac{dy}{dx} = 0$ gives $-12x + 12 = 0$ and $x = 1$

$y = 3e^{-2}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

**Answer: $(1, 3e^{-2})$, maximum**

## 5 Poisson over different intervals (CIE)

Faults are reported at random at an average of 10 per hour.
Find the probability that in a 90-minute period there are (a) zero events (b) at least 2.
(c) Find the probability that at least one occurs in each of two separate 90-minute periods.

$\lambda = 10 \times \frac{90}{60} = 15$, $X \sim Po(15)$

(a) $P(X = 0) = e^{-15} = 0.000000306$
(b) $P(X \geq 2) = 1 - P(X \leq 1) = 1.00$
(c) $(1 - 0.000000306)^2 = 1.00$
**Answer: (a) 0.000000306 (b) 1.00 (c) 1.00**

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## 6 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 4n^2 + 2n$.

(a) Find the first term and the common difference.

(b) Find $u_{30}$.

$u_1 = S_1 = 6$

$S_2 = 20$, so $u_2 = 14$

$d = u_2 - u_1 = 8$

$u_{30} = 6 + 29 \times 8$

**Answer: (a) a = 6, d = 8 (b) 238**

## 7 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 14\cot x + 2 = 0$

for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 14\cot x + 6 = 0$

$\cot x = 3$ or $\cot x = \frac{1}{2}$

$\cot x = 3$ means $\tan x = \frac{1}{3}$: $x = 18.4°$

$\cot x = \frac{1}{2}$ means $\tan x = 2$: $x = 63.4°$

**Answer: x = 18.4°, 63.4°**

## 8 Tangent conditions (All boards)

The line $y = k - 5x$ is a tangent to the curve $y = \frac{5}{x}$.

Find the possible values of k.

$\frac{5}{x} = k - 5x$

$5 = kx - 5x^2$, so $5x^2 - kx + 5 = 0$

Tangent: discriminant $k^2 - 4 \times 5 \times 5 = 0$

$k^2 = 100$

**Answer: k = 10 or k = −10**

## 9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 14$.

Assume $x^2 - y^2 = (x - y)(x + y) = 14$.

$x - y$ and $x + y$ are both odd or both even (they differ by 2y).

Both odd: the product is odd, but 14 is even.

Both even: the product is a multiple of 4, but 14 is not.

Contradiction either way.

**Answer: no such integers exist**

## 10 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(51, 6^2)$. Find $P(X > 56 \mid X > 53)$ to 3 significant figures.

Since $X > 56$ implies $X > 53$: $P = \frac{P(X > 56)}{P(X > 53)}$

$P(X > 56) = P(Z > 0.833) = 0.202$

$P(X > 53) = P(Z > 0.333) = 0.369$

$P = \frac{0.202}{0.369} = 0.548$

**Answer: 0.548**