Day 77 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 61 and standard deviation 5.
Find (a) P(X > 64) (b) P(64 < X < 74), each to 3 s.f.

(a) $z = \frac{64 - 61}{5} = 0.600$, $P(Z > 0.600) = 0.274$

(b) $z_1 = 0.600$, $z_2 = \frac{74 - 61}{5} = 2.60$

$P(64 < X < 74) = \Phi(2.60) - \Phi(0.600) = 0.270$

Answer: (a) 0.274 (b) 0.270

2 Differentiating a^x (All boards)

$y = 5 \times 3^x$.
The gradient of the curve at P is 135 ln 3. Find the x-coordinate of P.

$\frac{dy}{dx} = 5 \ln 3 \times 3^x$

$5 \ln 3 \times 3^x = 135 \ln 3$, so $3^x = 27$

$x = 3$, so $x = 3$

Answer: x = 3

3 Cubic inequalities (All boards)

Given that $(x - 5)$ is a factor of $f(x) = x^3 - 3x^2 - 25x + 75$, solve $f(x) > 0$.

Factorise: $f(x) = (x + 5)(x - 3)(x - 5)$

Roots at $x = -5, 3, 5$

Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)

Read off where $f(x) > 0$.

Answer: $-5 < x < 3$ or $x > 5$

4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 250 kg carries a person of mass 90 kg. The cable tension is 3604 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 340 kg), taking upwards as positive: $T - 340\,g = 340\,a$

$3604 - 3332 = 340\,a$, so $a = 0.8$

(a) a is positive, so the acceleration is 0.8 m s^{-2} upwards

(b) Person: $R - 90\,g = 90\,a$, so $R = 90(9.8 + 0.8)$

Answer: (a) 0.8 m s^{-2} upwards (b) 954 N

5 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$2\sin 3\theta + \cos 3\theta = 1.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos 3\theta \approx 1 - \frac{9}{2}\theta^2$

$2 \times 3\theta + 1(1 - \frac{9}{2}\theta^2) = 1.5$

Multiply by 2 and rearrange: $9\theta^2 - 12\theta + 1 = 0$

$\theta = \frac{12 \pm \sqrt{108}}{18}$, so $\theta = 0.0893$

Answer: a = 9, b = −12, c = 1; $\theta \approx 0.0893$

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6 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 49 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{3}{4}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8.

Horizontal component = 49×0.8 = 39.2; vertical component = 49×0.6 = 29.4
(a) Vertical: $0 = 29.4\,t - 4.9t^2$, so t = 6
(b) At the top: $0 = 29.4^2 - 2\times9.8\times H$, so H = 44.1
(c) OA = 39.2 × 6
Answer: (a) 6 s (b) 44.1 m (c) 235.2 m

7 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

30 values have $\Sigma x = 810$ and $\Sigma x^2 = 22620$.
One value was recorded as 39 but should have been 24.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 810 - 39 + 24 = 795$
$\Sigma x^2 = 22620 - 39^2 + 24^2 = 21675$
Mean $= \frac{795}{30} = 26.5$
sd $= \sqrt{\frac{21675}{30} - (26.5)^2} = 4.50$
Answer: mean 26.5, sd 4.50

8 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (2, 4). Its centre lies in the first quadrant.
Find the equations of the two possible circles.

Centre (r, r), radius r > 0.
Through (2, 4): $(2 - r)^2 + (4 - r)^2 = r^2$
$r^2 - 12r + 20 = 0$
(r − 2)(r − 10) = 0, so r = 2 or 10
Answer: $(x - 2)^2 + (y - 2)^2 = 4$ or $(x - 10)^2 + (y - 10)^2 = 100$

9 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = e^x$,

given that y = 2 when x = 0 (y > 0). Give y in terms of x.

Separate: $\int y\,dy = \int e^x\,dx$, so $\frac{y^2}{2} = e^x + C$
x = 0, y = 2 gives C = 1
$y^2 = 2e^x + 2$
Answer: $y = \sqrt{2e^x + 2}$

10 Modulus inequalities (All boards)

Solve $|2x - 7| > |-3x - 7|$.

Square both sides: $(2x - 7)^2 > (-3x - 7)^2$
$((2x - 7) - (-3x - 7))((2x - 7) + (-3x - 7)) > 0$
$(5x)(-x - 14) > 0$
Critical values x = −14 and x = 0; sketch the (positive or negative) quadratic.
Answer: $-14 < x < 0$