Day 76 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 5 m s^{-1} and accelerates uniformly. She travels 165 m in the next 15 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $165 = 5\times15 + \frac{1}{2}\times a\times15^2$, so $a = 0.8$
(b) $v = u + at = 5 + 0.8\times15 = 17$
(c) For the midpoint, $s = 82.5$: $v^2 = u^2 + 2as = 5^2 + 2\times0.8\times82.5$
$v^2 = 157$
Answer: (a) 0.8 m s^{-2} (b) 17 m s^{-1} (c) 12.5 m s^{-1}

2 Change of sign to locate a root (All boards)

$f(x) = e^x - 5x - 8$.
Show that the equation $f(x) = 0$ has a root α in the interval [3, 4].

f(x) is continuous on the interval
$f(3) = -2.91 < 0$
$f(4) = 26.6 > 0$
A change of sign, so a root lies in [3, 4]
Answer: Sign change: f(3) = −2.91, f(4) = 26.6

3 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{15}{8}$, $\mu = 0.5$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{15}{17}$, $\cos\alpha = \frac{8}{17}$, $R = 5g\cos\alpha = \frac{392}{17}$ N

Limiting friction $= \mu R = \frac{196}{17}$ N

P smallest: friction acts up the slope: $P + \mu R = 5g\sin\alpha$, $P = \frac{735}{17} - \frac{196}{17}$

P greatest: friction acts down the slope: $P = \frac{735}{17} + \frac{196}{17}$

Answer: 31.7 N ≤ P ≤ 54.8 N

4 Smallest whole number solving an exponential inequality (All boards)

Find the least integer n for which $3^n > 10^8$.

$n \lg 3 > 8$, so $n > \frac{8}{\lg 3} = 16.77$
Since n must be an integer, $n = 17$
Answer: n = 17

5 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB $= -\frac{2}{11}$, so gradient of the perpendicular $= \frac{11}{2}$

Through C: $11x - 2y = -25$

Line AB: $y - 3 = -\frac{2}{11}(x - -4)$. Solve to get D $= (-\frac{9}{5}, \frac{13}{5})$

Area $= \frac{1}{2} \times AB \times CD = 25$

Answer: (a) $11x - 2y = -25$ (b) D$(-\frac{9}{5}, \frac{13}{5})$, area 25

6 Separable differential equations (All boards)

Solve the differential equation

$(1 + x^2)\frac{dy}{dx} = xy$,

given that $y = 2$ when $x = 0$ ($y > 0$). Give y in terms of x.

Separate: $\int \frac{1}{y}\, dy = \int \frac{x}{1 + x^2}\, dx$, so $\ln y = \frac{1}{2}\ln(1 + x^2) + C$

$x = 0$, $y = 2$ gives $C = \ln 2$

$y = 2\sqrt{1 + x^2}$

Answer: $y = 2\sqrt{1 + x^2}$

7 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 2$, $y = 2$ so $a = 2$, $b = 2$

Through (5, 4): $4 = \frac{k}{3} + 2$

$k = (4 - 2)(3) = 6$

$x = 0$: $y = -1$; $y = 0$: $x = -1$

Answer: $k = 6$, $a = 2$, $b = 2$; (0, −1) and (−1, 0)

8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Parts are defective with probability 0.3, independently. A box of 10 parts is rejected if it has at least 4 defective. From 9 such boxes, find the probability that exactly 3 are rejected (3 s.f.).

$Y \sim B(10, 0.3)$. $P(\text{rejected}) = P(Y \geq 4) = 1 - 0.650 = 0.350$

$W \sim B(9, 0.350)$ is the number rejected

$P(W = 3) = C(9, 3)(0.350)^3(1 - 0.350)^6 = 0.272$

Answer: 0.272

9 Poisson over different intervals (CIE)

Customers enter a shop at random at an average of 10 per hour.
Find the probability that in a 120-minute period there are (a) zero events (b) at least 1.
(c) Find the probability that at least one occurs in each of two separate 120-minute periods.

$\lambda = 10 \times \frac{120}{60} = 20$, $X \sim Po(20)$

(a) $P(X = 0) = e^{-20} = 0.00000000206$

(b) $P(X \geq 1) = 1 - P(X \leq 0) = 1.00$

(c) $(1 - 0.00000000206)^2 = 1.00$

Answer: (a) 0.00000000206 (b) 1.00 (c) 1.00

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{13x + 1}{(1 + x)(1 - 3x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + x)} + \frac{B}{(1 - 3x)}$: $A = -3$, $B = 4$

(b) $-3(1 + x)^{-1} = -3(1 - x + 1x^2 \ldots)$

Add the series for both fractions.

(c) Need $|x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-3}{(1 + x)} + \frac{4}{(1 - 3x)}$ (b) $1 + 15x + 33x^2$ (c) $|x| < \frac{1}{3}$