**Day 75** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 29.4 m $s^{-1}$ from a point h m above horizontal ground, and hits the ground 8 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 29.4, a = −9.8, t = 8
$-h = 29.4\times8 - 4.9\times8^2$, so h = 78.4
(b) At the highest point v = 0: $0 = 29.4^2 - 2\times9.8\times s$
s = 44.1 m above the point of projection
Greatest height above the ground = 78.4 + 44.1
**Answer: (a) 78.4 m (b) 122.5 m**

## 2 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2\theta = -\sqrt{2}\cos\theta$ for $-180° \le \theta \le 180°$.

$\sin 2\theta \equiv 2\sin\theta\cos\theta$: $\cos\theta(2\sin\theta + \sqrt{2}) = 0$
$\cos\theta = 0$: $\theta = -90°, 90°$

$\sin\theta = \frac{-\sqrt{2}}{2}$: $\theta = -135°, -45°$

Do not divide by cos θ, as that would lose solutions
**Answer: $\theta = -135°, -90°, -45°, 90°$**

## 3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is normally distributed with mean 70 and standard deviation 12.
Find (a) P(X > 62.8) (b) P(62.8 < X < 78.4), each to 3 s.f.

(a) $z = \frac{62.8 - 70}{12} = -0.600$, $P(Z > -0.600) = 0.726$

(b) $z_1 = -0.600$, $z_2 = \frac{78.4 - 70}{12} = 0.700$
$P(62.8 < X < 78.4) = \Phi(0.700) - \Phi(-0.600) = 0.484$
**Answer: (a) 0.726 (b) 0.484**

## 4 Line meets curve: length of chord (All boards)

The line $y = -3x + 5$ meets the curve $y = x^2 - 4x - 15$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 - 4x - 15 = -3x + 5$ gives $x^2 - x - 20 = 0$
$(x + 4)(x - 5) = 0$: x = −4 or 5
A(−4, 17), B(5, −10)
$AB^2 = 9^2 + (-27)^2 = 810$
**Answer: A(−4, 17), B(5, −10); AB = $9\sqrt{10}$**

## 5 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg carries a person of mass 80 kg. The cable tension is 4896 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 480 kg), taking upwards as positive: T − 480 g = 480 a
4896 − 4704 = 480 a, so a = 0.4
(a) a is positive, so the acceleration is 0.4 m $s^{-2}$ upwards
(b) Person: R − 80 g = 80 a, so R = 80(9.8 + 0.4)
**Answer: (a) 0.4 m $s^{-2}$ upwards (b) 816 N**

## 6 Completing the square (All boards)

$f(x) = x^2 - 6x + 4$.

(a) Write $f(x)$ in the form $a(x + p)^2 + q$.

(b) State the minimum value of $f(x)$ and the value of $x$ at which it occurs.

(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = (x - 3)^2 - 5$

(b) minimum value $-5$ when $x = 3$

(c) $(x - 3)^2 = 5$, so $(x - 3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

**Answer: (a) $(x - 3)^2 - 5$ (b) minimum $-5$ at $x = 3$ (c) $x = 3 \pm \sqrt{5}$**

## 7 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 4 cm. Angle AOB = $\frac{\pi}{3}$.

Find the exact area and the exact perimeter of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = \frac{8\pi}{3}$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 4\sqrt{3}$

Segment area = sector − triangle = $\frac{8\pi - 12\sqrt{3}}{3}$

Chord AB = $2r\sin\frac{\theta}{2} = 4$; arc = $r\theta = \frac{4\pi}{3}$

Perimeter = arc + chord = $\frac{4\pi + 12}{3}$

**Answer: area = $\frac{8\pi - 12\sqrt{3}}{3}$ cm$^2$, perimeter = $\frac{4\pi + 12}{3}$ cm**

## 8 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(38, p)$. $H_0$: $p = 0.25$ is tested against $H_1$: $p \neq 0.25$ at the 10% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail $\leq 0.05$. $P(X \leq 4) = 0.0235$, $P(X \leq 5) = 0.0604$

$P(X \geq 15) = 0.0348$, $P(X \geq 14) = 0.0710$

Critical region: $X \leq 4$ or $X \geq 15$

(b) Actual level = 0.0235 + 0.0348 = 0.0583

**Answer: (a) $X \leq 4$ or $X \geq 15$ (b) 0.0583**

## 9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - x)^{\frac{1}{2}}$ in ascending powers of $x$ up to $x^2$.

(b) Use $x = 0.02$ to estimate $\sqrt{98}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -1x$

(a) $1 - \frac{1}{2}x - \frac{1}{8}x^2$

$x = \frac{1}{50}$: $98 = 10^2(1 - \frac{1}{50})$ so $\sqrt{98} = 10 \times (1 - \frac{1}{50})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{100} - \frac{1}{20000}) = 9.899500$

**Answer: (a) $1 - \frac{1}{2}x - \frac{1}{8}x^2$ (b) 9.8995**

## 10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(−2, 0, 0) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} t-1 \\ 1 \\ t-1 \end{pmatrix}$

PF·d = 0 gives $2t - 2 = 0$, so $t = 1$

F = (−2, 1, 0); $\mathrm{PF} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{1} = 1$

**Answer: F(−2, 1, 0), distance 1**