Day 74 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−3, 11) and B(−3, −15) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(2, 10) lies on the circle.

Centre = midpoint of AB = (−3, −2)
$r^2 = (0)^2 + (13)^2 = 169$
$(x + 3)^2 + (y + 2)^2 = 169$: $x^2 + y^2 + 6x + 4y - 156 = 0$
(b) $CM^2 = (5)^2 + (12)^2 = 169 = r^2$
Answer: (a) $x^2 + y^2 + 6x + 4y - 156 = 0$

2 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 40 members is taken from three groups.
13 of the sample are from Bronze, which has 65 members.
(a) Find the total number of members.
(b) Silver has 25 members. How many are sampled?

Bronze sampled fraction = $\frac{13}{65} = \frac{1}{5}$

Total = $40 \div \frac{1}{5} = 200$

Silver: $25 \times \frac{1}{5} = 5$

Answer: (a) 200 (b) 5

3 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 4, |b| = 2 and the angle between a and b is 60°.
Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 60° = 4 \times 2 \times \frac{1}{2} = 4$

$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 16 + 8 + 4 = 28$
$|\mathbf{a} + \mathbf{b}| = 2\sqrt{7}$
Answer: $\mathbf{a}\cdot\mathbf{b} = 4$, $|\mathbf{a} + \mathbf{b}| = 2\sqrt{7}$

4 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 10 m s^{-1} from a point 19.6 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $19.6 = \frac{1}{2} \times 9.8 \times t^2$, so t = 2
(a) Horizontal distance = 10 × 2
(b) Vertical speed on landing = 9.8×2 = 19.6; horizontal speed = 10
Speed = $\sqrt{(10^2 + 19.6^2)} = 22.0$
$\tan\theta = 19.6 \div 10$, $\theta = 63.0°$
Answer: (a) 20 m (b) 22.0 m s^{-1} at 63.0° below the horizontal

5 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{5}{\sqrt{2} - 1} + \frac{4}{\sqrt{2} + 1}$ can be written as $p + q\sqrt{2}$, and find p and q.

Common denominator $(\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1$
Numerator = $5(\sqrt{2} + 1) + 4(\sqrt{2} - 1)$
$= 1 + 9\sqrt{2}$
Divide by 1.
Answer: p = 1, q = 9

6 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^3 x\, e^{-2x}\, dx$.

$u = x$, $\frac{dv}{dx} = e^{-2x}$: $\int xe^{-2x}\, dx = [-\frac{xe^{-2x}}{2}] + \int \frac{e^{-2x}}{2}\, dx$

$= [-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}]$

$= \frac{1 - 7e^{-6}}{4}$

Answer: $\mathbf{\frac{1 - 7e^{-6}}{4}}$

7 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The life of a battery is $N(54, 5^2)$. A battery is classed as 'special' if its life is less than 63. 8 batterys are chosen at random. Find the probability that exactly 4 are special (3 s.f.).

$P(special) = P(X < 63) = P(Z < 1.80) = 0.964$

$Y \sim B(8, 0.964)$

$P(Y = 4) = C(8, 4)(0.964)^4(0.0359)^4 = 0.000101$

Answer: 0.000101

8 Tangent through the origin (All boards)

The tangent to the curve $y = \ln(5x)$ at the point P passes through the origin. Find the exact coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = \frac{1}{x}$; tangent at $x = a$: $y - \ln(5a) = \frac{1}{a}(x - a)$

Through (0, 0): $-\ln(5a) = -1$, so $5a = e$ and $a = \frac{e}{5}$

$P = (\frac{e}{5}, 1)$, gradient $\frac{1}{a} = 5e^{-1}$, so the tangent is $y = 5e^{-1}x$

Answer: $\mathbf{P(\frac{e}{5}, 1)}$**; y =** $\mathbf{5e^{-1}x}$

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{x - 3}{(1 + 3x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 3x)} + \frac{B}{(1 + x)}$: $A = -5$, $B = 2$

(b) $-5(1 + 3x)^{-1} = -5(1 - 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need $|3x| < 1$ and $|x| < 1$

Answer: (a) $\mathbf{\frac{-5}{(1 + 3x)} + \frac{2}{(1 + x)}}$ **(b)** $\mathbf{-3 + 13x - 43x^2}$ **(c)** $\mathbf{|x| < \frac{1}{3}}$

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(-3\mathbf{i} - 8\mathbf{j})$ m and $(3\mathbf{i} + 4\mathbf{j})$ m and velocities $(-\mathbf{i} + 5\mathbf{j})$ m s^{-1} and $(2\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$PQ = r_P - r_Q = (-6\mathbf{i} - 12\mathbf{j}) + t(-\mathbf{i} + 3\mathbf{j})$

$d^2 = (-t - 6)^2 + (3t - 12)^2 = 10t^2 - 60t + 180$

(a) $d(d^2)/dt = 0$: $20t + -60 = 0$, so $t = 3$

(b) At t = 3: $PQ = -9\mathbf{i} - 3\mathbf{j}$, so $d = \sqrt{90}$

Answer: (a) t = 3 s (b) 9.49 m