Day 73 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

f(x) = 4x + 5 and g(x) = x^2 + 6.
Solve gf(x) = 7.

gf(x) = g(4x + 5) = $(4x + 5)^2$ + 6
$(4x + 5)^2$ + 6 = 7
4x + 5 = ±1

Answer: x = $-\frac{3}{2}$ or x = −1

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2 x - 5\sin x + 1 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x \equiv 1 - \sin^2 x$: $2\sin^2 x + 5\sin x - 3 = 0$

Solving the quadratic: $\sin x = \frac{1}{2}$ or $\sin x = -3$

$\sin x = \frac{1}{2}$: x = 30°, 150°

$\sin x = -3$ has no solutions ($|\sin x| \leq 1$)

Answer: x = 30°, 150°

3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1470 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 95) × V
1470 = 122.5 V, so V = 12
(b) Deceleration = V ÷ 25 = 12 ÷ 25

Answer: (a) V = 12 (b) 0.48 m s^{-2}

4 Foot of a perpendicular (All boards)

The line l has equation x + 2y = −1.
(a) Find the coordinates of the foot of the perpendicular from P(1, −3) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = $-\frac{1}{2}$, so perpendicular gradient = 2

Perpendicular through P: 2x − y = 5

Solve with l: x = $\frac{9}{5}$, y = $-\frac{7}{5}$

Distance^2 = $(1 - \frac{9}{5})^2 + (-3 - (-\frac{7}{5}))^2 = \frac{16}{5}$

Answer: (a) $(\frac{9}{5}, -\frac{7}{5})$ (b) $\frac{4\sqrt{5}}{5}$

5 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 5 kg is pulled at constant speed along rough horizontal ground, $\mu = \frac{1}{3}$, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: R = 5 g − P sin θ

Horizontal: P cos θ = μR = $\frac{1}{3}$(5 g − P sin θ)

$P(\frac{5}{13} + \frac{4}{13}) = \frac{1}{3} \times 5\ g$

P = 23.6, R = 27.2

Answer: P = 23.6 N, R = 27.2 N

6 Smallest whole number solving an exponential inequality (All boards)

Find the least integer n for which $3^n > 10^9$.

$n \lg 3 > 9$, so $n > \frac{9}{\lg 3} = 18.86$

Since n must be an integer, n = 19

Answer: n = 19

7 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \times 3^{2x} - 10 \times 3^x + 3 = 0$.

Let $y = 3^x$. Then $3y^2 - 10y + 3 = 0$

$3(y - \frac{1}{3})(y - 3) = 0$

$y = \frac{1}{3}$ or $y = 3$

$3^x = \frac{1}{3}$ gives $x = -1$; $3^x = 3$ gives $x = 1$

Answer: x = −1 or x = 1

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{1}{8}(4 - x)$ for $0 \leq x \leq 4$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \leq x \leq 4$.

(b) Find P(X > 2). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{2x}{4} - \frac{x^2}{16}$

(b) $P(X > 2) = 1 - F(2) = \frac{1}{4}$

(c) $F(m) = \frac{1}{2}$: $\frac{2m}{4} - \frac{m^2}{16} = \frac{1}{2}$, so $m = 4(1 - \frac{1}{\sqrt{2}})$

m = 1.17

Answer: (b) $\frac{1}{4}$ (c) 1.17

9 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 9 pairs of (x, y): $\Sigma x = 124$, $\Sigma y = 234$, $\Sigma x^2 = 2590$, $\Sigma xy = 4064$.

(a) Find the equation of the regression line of y on x (3 s.f.).

(b) Estimate y when x = 13.

$S_{xx} = 2590 - \frac{124^2}{9} = 882$, $S_{xy} = 4064 - \frac{124 \times 234}{9} = 840$

$b = \frac{S_{xy}}{S_{xx}} = 0.953$

$a = \bar{y} - b\,\bar{x} = 26.0 - (0.953)(13.8) = 12.9$

(b) $y = 12.9 + (0.953)(13) = 25.3$

Answer: (a) y = 12.9 + 0.953x (b) 25.3

10 Product rule: stationary points (All boards)

$y = (3x - 6)e^{-x}$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 3e^{-x} - (3x - 6)e^{-x} = (-3x + 9)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-3x + 9 = 0$ and $x = 3$

$y = 3e^{-3}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(3, 3e^{-3})$, maximum