Day 72 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Change of sign to locate a root (All boards)

$f(x) = \ln x + 4x - 6$.

Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$.

$f(x)$ is continuous on the interval

$f(1) = -2.00 < 0$

$f(2) = 2.69 > 0$

A change of sign, so a root lies in $[1, 2]$

Answer: Sign change: $f(1) = -2.00$, $f(2) = 2.69$

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(1 - 2x)^9$.

Term in x^2: ${}^9C_2 \times 1^7 \times (-2x)^2$

$= 36 \times 1 \times 4$

Answer: 144

3 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled along rough horizontal ground, $\mu = 0.2$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{12}{5}$.

The tension is 40 N. Find (a) the normal reaction, (b) the acceleration of the box. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

(a) Vertical: $R + 40\sin\theta = 4g$, so $R = \frac{148}{65}$

(b) Horizontal: $40\cos\theta - \mu R = 4a$

$\frac{200}{13} - 0.2 \times \frac{148}{65} = 4a$

$a = \frac{1213}{325}$

Answer: (a) 2.28 N (b) 3.73 m s^{-2}

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For $n = 8$ pairs of (x, y): $\Sigma x = 148$, $\Sigma y = 155$,

$\Sigma x^2 = 3062$, $\Sigma y^2 = 3661$, $\Sigma xy = 2520$.

Calculate the product moment correlation coefficient, and describe the correlation.

$S_{xx} = 3062 - \frac{148^2}{8} = 324$

$S_{yy} = 3661 - \frac{155^2}{8} = 658$

$S_{xy} = 2520 - \frac{148 \times 155}{8} = -348$

$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = -0.753$

This is strong negative correlation

Answer: $r = -0.753$, negative correlation

5 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 48 m ahead. After a reaction time of 0.4 s she brakes with deceleration 5 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance $= 0.4v$, braking distance $= v^2 \div (2 \times 5)$

$0.4v + v^2 \div 10 = 48$, so $v^2 + 4v - 480 = 0$

$(v - 20)(v + 24) = 0$, so $v = 20$

(b) Braking time $= 20 \div 5 = 4$ s; total $= 0.4 + 4$

Answer: (a) 20 m s^{-1} (b) 4.4 s

6 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 values have $\Sigma x = 600$ and $\Sigma x^2 = 18720$.
One value was recorded as 46 but should have been 32.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 600 - 46 + 32 = 586$

$\Sigma x^2 = 18720 - 46^2 + 32^2 = 17628$

Mean $= \frac{586}{20} = 29.3$

sd $= \sqrt{\frac{17628}{20} - (29.3)^2} = 4.79$

Answer: mean 29.3, sd 4.79

7 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^2 in the expansion of
$(2 + 2x)(1 + x)^5$.

In $(1 + x)^5$: coefficient of x^1 is 5, of x^2 is 10
Coefficient $= 2 \times 10 + 2 \times 5$
Answer: 30

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $-4 + 3\lambda = -1 + 3\mu$; $2 + 1\lambda = 1 + 2\mu$; $-8 + 2\lambda = -4 + 0\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)

9 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2\,\theta - 14\cot\theta + 22 = 0$
for $0° < \theta < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2\,\theta \equiv 1 + \cot^2\theta$: $2\cot^2\theta - 14\cot\theta + 24 = 0$
$\cot\theta = 3$ or $\cot\theta = 4$

$\cot\theta = 3$ means $\tan\theta = \frac{1}{3}$: $\theta = 18.4°, 198.4°$

$\cot\theta = 4$ means $\tan\theta = \frac{1}{4}$: $\theta = 14.0°, 194.0°$

Answer: θ = 14.0°, 18.4°, 194.0°, 198.4°

10 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(2, 4) and B(−8, 4) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(1, 1) lies on the circle.

Centre = midpoint of AB = (−3, 4)
$r^2 = (5)^2 + (0)^2 = 25$
$(x + 3)^2 + (y - 4)^2 = 25$: $x^2 + y^2 + 6x - 8y = 0$
(b) $CM^2 = (4)^2 + (-3)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 + 6x - 8y = 0$