Day 71 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(2 + 3x)^5$.

Term in x^4: ${}^5C_4 \times 2^1 \times (3x)^4$

$= 5 \times 2 \times 81$

Answer: 810

2 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 42.5 m above the sea at 20 m s^{-1}, at angle α above the horizontal, $\tan\alpha = \frac{4}{3}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 12, vertical component 16 (upwards)

(a) $-42.5 = 16\,t - 4.9t^2$, so $4.9t^2 - 16\,t - 42.5 = 0$

$t = [16 + \sqrt{1089}] \div 9.8 = 5$ (taking the positive root)

(b) Distance = 12×5

(c) Vertical speed = $16 - 9.8\times5 = -33$; speed = $\sqrt{(12^2 + 33^2)}$

Answer: (a) 5 s (b) 60 m (c) 35.1 m s^{-1}

3 Sigma notation (All boards)

Find the value of $\sum_{r=1}^{6} 5 \times \frac{3}{2}^r$.

Geometric series: first term $\frac{15}{2}$, ratio $\frac{3}{2}$, 6 terms

$$S = \frac{\frac{15}{2}(\frac{3}{2}^6 - 1)}{(\frac{3}{2} - 1)}$$

$$= \frac{\frac{15}{2} \times \frac{665}{64}}{\frac{1}{2}}$$

Answer: $\frac{9975}{64}$

4 Differentiating sec, cosec, tan and cot (All boards)

y = cosec(2x).

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{12}$.

$\frac{dy}{dx} = -2\text{cosec}(2x)\cot(2x)$

At $x = \frac{\pi}{12}$ the angle $2x = \frac{\pi}{6}$, so the gradient is $-4\sqrt{3}$

Answer: $\frac{dy}{dx} = -2\text{cosec}(2x)\cot(2x)$; $-4\sqrt{3}$

5 Arithmetic series: finding n (All boards)

An arithmetic series has first term 8 and common difference 6.
The sum of the first n terms is 848. Find n.

$\frac{n}{2}(2 \times 8 + (n - 1) \times 6) = 848$

$6n^2 + 10n = 1696$

$6n^2 + 10n - 1696 = 0$

$(n - 16)(6n + 106) = 0$, so $n = 16$ or $n = -\frac{53}{3}$

n must be a positive integer, so n = 16

Answer: 16

6 Smooth incline held by a string (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle rests on a smooth plane inclined at α, $\tan\alpha = \frac{4}{3}$, held by a string in the vertical plane through a line of greatest slope, at angle θ above the slope, $\tan\theta = \frac{5}{12}$. Find the tension and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

Along the plane: $T\cos\theta = 5g\sin\alpha$, so $T = 42.5$

Perpendicular: $R + T\sin\theta = 5g\cos\alpha$

$R = 5g\cos\alpha - T\sin\theta = 29.4 - 42.5\times0.385$

Answer: T = 42.5 N, R = 13.1 N

7 Stationary points with trigonometric and exponential terms (All boards)

$y = x - 2\sin x$ for $0 \le x \le 2\pi$.

Find the exact coordinates of the stationary points and determine their nature.

$\frac{dy}{dx} = 1 - 2\cos x = 0$, so $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$ in the range

$\frac{d^2y}{dx^2} = 2\sin x$: sign at each point gives the nature

Answer: $\left(\frac{\pi}{3}, \frac{\pi - 3\sqrt{3}}{3}\right)$ minimum; $\left(\frac{5\pi}{3}, \frac{3\sqrt{3} + 5\pi}{3}\right)$ maximum

8 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{6}{64}x(4 - x)$ for $0 \le x \le 4$, and 0 otherwise.

Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^4 x\, f(x)\, dx = 2$

$E(X^2) = \int_0^4 x^2\, f(x)\, dx = \frac{24}{5}$

$\text{Var}(X) = \frac{24}{5} - (2)^2 = \frac{4}{5}$

Standard deviation $= \sqrt{\frac{4}{5}} = 0.894$

Answer: E(X) = 2, Var(X) = $\frac{4}{5}$, sd 0.894

9 Equations with cosec x and cot x (All boards)

Solve $2\text{cosec}^2 x + 5\cot x - 5 = 0$

for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $2\cot^2 x + 5\cot x - 3 = 0$

$\cot x = \frac{1}{2}$ or $\cot x = -3$

$\cot x = \frac{1}{2}$ means $\tan x = 2$: $x = 63.4°$

$\cot x = -3$ means $\tan x = -\frac{1}{3}$: $x = 161.6°$

Answer: x = 63.4°, 161.6°

10 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 33, mean 37, sd 5;
group Y has n = 27, mean 57, sd 3.
Find the mean and standard deviation of all 60 values (3 s.f. for the sd).

$\Sigma x = 33 \times 37 + 27 \times 57 = 2760$, mean = 46

Σx^2 (A) $= 33(5^2 + 37^2) = 46002$

Σx^2 (B) $= 27(3^2 + 57^2) = 87966$

Variance $= \frac{133968}{60} - 46^2 = 116.8$, so sd = 10.8

Answer: mean 46, sd 10.8