Day 68 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Sigma notation (All boards)

Evaluate $\sum_{r=7}^{29}(5r-3)$.

Arithmetic series: first term 32, last term 142, 23 terms

$S = \frac{23}{2}(32 + 142) = 2001$

Answer: 2001

2 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 8 pairs of (x, y): $\Sigma x = 107$, $\Sigma y = 430$, $\Sigma x^2 = 1661$, $\Sigma xy = 6459$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 17.

$S_{xx} = 1661 - \frac{107^2}{8} = 230$, $S_{xy} = 6459 - \frac{107 \times 430}{8} = 708$

$b = \frac{S_{xy}}{S_{xx}} = 3.08$

$a = \bar{y} - b\,\bar{x} = 53.8 - (3.08)(13.4) = 12.6$

(b) $y = 12.6 + (3.08)(17) = 64.9$

Answer: (a) $y = 12.6 + 3.08x$ (b) 64.9

3 Chain rule and gradients (All boards)

$y = (\ln x)^4$ for $x > 0$.

Find $\frac{dy}{dx}$ and its exact value at x = e.

Let $u = \ln x$: $y = u^4$, so $\frac{dy}{dx} = 4u^3 \times \frac{1}{x} = \frac{4(\ln x)^3}{x}$

At x = e: $\ln x = 1$, so gradient $= \frac{4}{e}$

Answer: $\frac{dy}{dx} = \frac{4(\ln x)^3}{x}$; gradient $= \frac{4}{e}$

4 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{-1} in the expansion of

$(x - \frac{2}{x^2})^5$.

General term: ${}^5C_r\,(x^1)^{5-r}\,(-2x^{-2})^r$

Power of x: $1(5 - r) - 2r = -1$

$5 - 3r = -1$ so $r = 2$

Coefficient $= {}^5C_2 \times 1^3 \times (-2)^2 = 40$

Answer: 40

5 Work-energy on a rough slope (CIE)

A 6 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.5$, and slides 10 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 10 m. Take g = 10.

$R = 6g\cos\alpha = 36$ N, $F = \mu R = 18$ N

(a) Work done against friction $= F \times d = 18 \times 10$

(b) Loss of PE $= 6g \times 10\sin\alpha = 480$ J

Work-energy: $\frac{1}{2} \times 6 \times v^2 = 480 - 180 = 300$

$v^2 = 100$

Answer: (a) 180 J (b) 10 m s^{-1}

6 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 + 5$, $y = 2t(t^2 - 4)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 4$, so $t = 0, 2, -2$
$t = 0$: $x = 5$
$t = \pm 2$: $x = 13$
Answer: (5, 0) and (13, 0)

7 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (4 kg, 2.5 m) is smoothly hinged at A to a vertical wall, with a 5 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{3}{4}$. Find (a) the tension, (b) the magnitude of the hinge force. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Moments about A: $T \sin\theta \times 2.5 = 4g \times 1.25 + 5g \times 2.5$
$T \sin\theta = (2 + 5)g = 68.6$, so (a) $T = 114$
Horizontal component of hinge force: $X = T\cos\theta = 91.5$
Vertical: $Y + T\sin\theta = (4 + 5)g$, so $Y = 4g \div 2 = 19.6$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(91.5^2 + 19.6^2)}$
Answer: (a) 114 N (b) 93.5 N

8 Partial fractions and integration (All boards)

(a) Express $\frac{x + 7}{(x + 4)^2}$ in the form $\frac{A}{x + 4} + \frac{B}{(x + 4)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{x + 7}{(x + 4)^2}\,dx$.

$x + 7 \equiv A(x + 4) + B$, so $A = 1$ and $B = 3$

$I = [1\ln(x + 4) - \frac{3}{x + 4}]_0^4$

$= 1\ln\frac{8}{4} + 3(\frac{1}{4} - \frac{1}{8})$

$= \frac{8\ln 2 + 3}{8}$

Answer: (a) A = 1, B = 3; (b) $\frac{8\ln 2 + 3}{8}$

9 Hypothesis test for a Poisson mean (CIE)

Complaints occur as a Poisson process at a mean rate of 6 per day.
After a change, the number recorded in one day is 12.
Test at the 10% level whether the mean rate has increased.

H_0: $\lambda = 6$, H_1: $\lambda > 6$; $X \sim Po(6)$
$P(X \geq 12) = 1 - P(X \leq 11) = 0.0201$
$0.0201 < 0.1$
Reject H_0: evidence the rate has increased.
Answer: p-value 0.0201; reject H_0

10 Logarithmic equations (All boards)

Solve $\log_2(x + 6) + \log_2(x - 2) = 7$.

Combine: $\log_2[(x + 6)(x - 2)] = 7$, so $(x + 6)(x - 2) = 2^7 = 128$
$x^2 + 4x - 140 = 0$: $(x - 10)(x + 14) = 0$
$x = 10$ or $x = -14$; $x = -14$ makes $x - 2$ negative, so it is rejected
Answer: x = 10