Day 66 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-3x + 4y = k$ is a tangent to the circle
$(x - 1)^2 + (y - 3)^2 = 9$.
Find the two possible values of k.

Centre (1, 3), radius 3
Distance from centre to the line = radius:
$\frac{|9 - k|}{5} = 3$
$|9 - k| = 15$
Answer: $k = -6$ or $k = 24$

2 Index equations solved with logarithms (All boards)

Solve $7^{2x-3} = 2^{3x-1}$.
Give your answer to 3 significant figures.

Take logs: $(2x - 3)\ln 7 = (3x - 1)\ln 2$
Collect x terms: $x(2\ln 7 - 3\ln 2) = -\ln 2 + 3\ln 7$
$x = \frac{-\ln 2 + 3\ln 7}{2\ln 7 - 3\ln 2}$
$x = 2.84$
Answer: $x = 2.84$

3 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 2 kg particle moves on a smooth horizontal line. At time t s a single force of $(16 - 4t)$ N acts along the line. When $t = 0$ its speed is 2 m s^{-1}. Find (a) its speed when $t = 2$, (b) its greatest speed.

$F = ma$: $a = (-4t + 16) \div 2$
$v = \int a\,dt = 8t - 1t^2 + c$; $v = 2$ when $t = 0$
(a) $v(2) = 14$
(b) Greatest speed when $a = 0$: $t = 4$
$v(4) = 18$
Answer: (a) 14 m s^{-1} (b) 18 m s^{-1}

4 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $n^2 + n + 9$ is odd for every positive integer n.

$n^2 + n + 9 = n(n + 1) + 9$
$n(n + 1)$ is the product of consecutive integers, so it is even.
Even + 9 (odd) = odd.
Answer: $n(n + 1)$ is even, so $n^2 + n + 9$ is odd

5 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{24}{25}$.
Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos\theta = -\sqrt{1 - \left(\frac{24}{25}\right)^2} = -\frac{7}{25}$ (obtuse)
$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{24}{25} \times \left(-\frac{7}{25}\right) = -\frac{336}{625}$
$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \left(\frac{24}{25}\right)^2 = -\frac{527}{625}$
$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{336}{527}$
Answer: $\sin 2\theta = -\frac{336}{625}$, $\cos 2\theta = -\frac{527}{625}$, $\tan 2\theta = \frac{336}{527}$

6 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is N(80, 2^2).
Find the symmetric limits about the mean between which the central 98% of values lie (3 s.f.).

Each tail has area 0.01, so P(Z < z) = 0.99
z = 2.33
Limits: 80 ± 2.33 × 2
= 75.3 and 84.7
Answer: 75.3 to 84.7

7 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
5sin 2θ + 2cos 2θ = 3.5
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 2\theta \approx 2\theta$, $\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$

$5 \times 2\theta + 2(1 - \frac{4}{2}\theta^2) = 3.5$

Multiply by 2 and rearrange: $8\theta^2 - 20\theta + 3 = 0$

$\theta = \frac{20 \pm \sqrt{304}}{16}$, so $\theta = 0.160$

Answer: a = 8, b = −20, c = 3; $\theta \approx 0.160$

8 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 22 pairs of values of hours of sunshine and plant height, r = −0.322.
The critical value for a two-tailed test at the 5% level is 0.4227.
Test, at this level, whether there is any correlation between the two variables.

H_0: ρ = 0, H_1: ρ ≠ 0
Critical value 0.4227
|r| = 0.322 < 0.4227
This is not in the critical region, so do not reject H_0.
There is insufficient evidence of correlation.
Answer: Do not reject H_0: insufficient evidence

9 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations x = 3t − 1, $y = 4t^2$.
Find a Cartesian equation of the curve.

From x: $t = \frac{x + 1}{3}$

$y = 4(\frac{x + 1}{3})^2$

Expand.

Answer: $y = \frac{4}{9}x^2 + \frac{8}{9}x + \frac{4}{9}$

10 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 6 g sin 60°; B: 4 g sin 60°
A moves down, B up (6 sin 60° > 4 sin 60°): F = ma for the system
6 g sin 60° − 4 g sin 60° = (6 + 4)a
(a) a = 1.70
(b) For A: 6 g sin 60° − T = 6 a, so T = 40.7
Answer: (a) 1.70 m s^{-2}, A moves down (b) 40.7 N