Day 65 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 6x - 6y - 82 = 0$ passes through P(−11, 9).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−3, 3). Gradient of radius CP = $-\frac{3}{4}$

Gradient of tangent = $\frac{4}{3}$

$y - 9 = \frac{4}{3}(x + 11)$: $4x - 3y = -71$

y = 0: $x = -\frac{71}{4}$

Answer: (a) $4x - 3y = -71$ (b) $(-\frac{71}{4}, 0)$

2 Venn diagrams and conditional probability (All boards)

P(A) = 0.45, P(B) = 0.65 and P(A ∩ B) = 0.15.
Find
(a) P(A ∪ B)
(b) P(A | B′)
(c) P(A′ ∩ B′)

(a) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.45 + 0.65 − 0.15 = 0.95
(b) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.45 - 0.15}{1 - 0.65} = \frac{6}{7}$
(c) P(A′ ∩ B′) = 1 − P(A ∪ B) = 1 − 0.95 = 0.05

Answer: (a) 0.95 (b) $\frac{6}{7}$ (c) 0.05

3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 2040 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(100 + 70) × V
2040 = 85 V, so V = 24
(b) Deceleration = V ÷ 15 = 24 ÷ 15

Answer: (a) V = 24 (b) 1.6 m s^{-2}

4 Conditional probability and independence (All boards)

P(B) = 0.7, P(A) = 0.3 and P(A | B) = 0.4.
(a) Find P(A ∩ B) and P(A ∪ B).
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) P(A ∩ B) = P(A | B)P(B) = 0.4 × 0.7 = 0.28
P(A ∪ B) = 0.3 + 0.7 − 0.28 = 0.72
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.28}{0.3} = \frac{14}{15}$
(c) P(A)P(B) = 0.3 × 0.7 = 0.21, which does not equal P(A ∩ B) = 0.28
So A and B are not independent

Answer: (a) 0.28, 0.72 (b) $\frac{14}{15}$ (c) not independent

5 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 2) = -5 - 3\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$x = \dfrac{-5 - 3\sqrt{2}}{\sqrt{2} - 2}$

Multiply top and bottom by $\sqrt{2} + 2$.

Numerator: $(-5 - 3\sqrt{2})(\sqrt{2} + 2) = -16 - 11\sqrt{2}$

Denominator: $2 - 4 = -2$

Answer: $x = 8 + \frac{11}{2}\sqrt{2}$

6 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{8}{17}$ and $\cos B = -\frac{7}{25}$.

Find the exact values of cos(A−B) and cos(A+B).

$\cos A = \frac{15}{17}$ (A acute), $\sin B = \frac{24}{25}$ (B obtuse), so $\tan A = \frac{8}{15}$, $\tan B = -\frac{24}{7}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B = \frac{15}{17} \times (-\frac{7}{25}) + \frac{8}{17} \times \frac{24}{25} = \frac{87}{425}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{15}{17} \times (-\frac{7}{25}) - \frac{8}{17} \times \frac{24}{25} = -\frac{297}{425}$

Answer: $\cos(A-B) = \frac{87}{425}$, $\cos(A+B) = -\frac{297}{425}$

7 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Show by counter-example that the statement "$2^n - 1$ is prime for all integers $n \geq 2$" is false.

Try n = 4:

$2^4 - 1 = 15 = 3 \times 5$

This is not prime.

Answer: n = 4 gives $15 = 3 \times 5$

8 Greatest and least values using R-form (All boards)

$g(\theta) = \dfrac{1}{24 + 8\sin\theta + 15\cos\theta}$

(a) Write $8\sin\theta + 15\cos\theta$ in the form $R\sin(\theta + \alpha)$, with α acute.

(b) Find the greatest and least values of $g(\theta)$ and the smallest positive θ at which the greatest value occurs (1 d.p.).

$R = \sqrt{8^2 + 15^2} = 17$, $\alpha = \tan^{-1}\frac{15}{8} = 61.9°$

Denominator $= 24 + 17\sin(\theta + 61.9°)$, ranging from 7 to 41

Greatest $g = \frac{1}{7}$ (denominator least), least $g = \frac{1}{41}$

Least denominator when $\sin(\theta + 61.9°) = -1$: $\theta + 61.9° = 270°$

$\theta = 208.1°$

Answer: R = 17, α = 61.9°; greatest $\frac{1}{7}$, least $\frac{1}{41}$; θ = 208.1°

9 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 6$.

Find the gradients of the two possible tangents and the points of contact.

Tangent at x = a: $y - (a^2 + 6) = 2a(x - a)$. Through (0, 0): $-a^2 - 6 = -2a^2$

So $a^2 = 6$ and $a = \pm\sqrt{6}$

Gradient $2a = \pm 2\sqrt{6}$

Answer: gradients $\pm 2\sqrt{6}$ at points $(\pm\sqrt{6}, 12)$

10 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$; friction $F = \mu R = 0.25\times 3\,g\cos\alpha$

B: $5g - T = 5a$

A: $T - 3g\sin\alpha - F = 3a$

Adding: $g(5 - \frac{36}{13} - \frac{15}{52}) = 8a$

(a) $a = 2.38$; (b) $T = 5(g - a)$

Answer: (a) 2.38 m s^{-2} (b) 37.1 N