Day 64 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 296 items in a list are numbered 1 to 296.
A systematic sample of 37 items is taken, starting with a random item among the first few.
The first item selected is number 7.
Find the sampling interval and the number of the last item selected.

Interval = 296 ÷ 37 = 8
Items selected: 7, 15, 23, ...
Last = 7 + (37 − 1) × 8 = 295
Answer: Interval 8, last item 295

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 8 m s^{-1} and accelerates uniformly. She travels 255 m in the next 15 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $255 = 8\times15 + \frac{1}{2}\times a\times15^2$, so $a = 1.2$
(b) $v = u + at = 8 + 1.2\times15 = 26$
(c) For the midpoint, $s = 127.5$: $v^2 = u^2 + 2as = 8^2 + 2\times1.2\times127.5$
$v^2 = 370$
Answer: (a) 1.2 m s^{-2} (b) 26 m s^{-1} (c) 19.2 m s^{-1}

3 Poisson probabilities (CIE)

Calls reach a switchboard at a mean rate of 2 per hour, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 5 (b) at least 5 (c) fewer than 5 (3 s.f.).

X ~ Po(2)
(a) $P(X = 5) = e^{-2}\,2^5/120! = 0.0361$
(b) $1 - P(X \leq 4) = 1 - 0.947 = 0.0527$
(c) $P(X \leq 4) = 0.947$
Answer: (a) 0.0361 (b) 0.0527 (c) 0.947

4 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 6 that lie strictly between 30 and 466.

First multiple 36, last multiple 462, common difference 6.
Number of terms $n = \frac{462 - 36}{6} + 1 = 72$
$S = \frac{72}{2}(36 + 462) = 17928$
Answer: 17928

5 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

A circular ripple has area A cm^2 increasing at a constant 3 cm^2s^{-1}.
Find the rate at which the circumference is increasing when the radius is 5 cm.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dt} = \frac{3}{2\pi r}$
$C = 2\pi r$, so $\frac{dC}{dt} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{3}{2\pi \times 5} = \frac{3}{5}$
Answer: $\frac{3}{5}$ cm/s

6 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^4 in the expansion of
$(3 + 2x)(2 - 2x)^5$.

In $(2 - 2x)^5$: coefficient of x^3 is −320, of x^4 is 160
Coefficient = 3 × 160 + 2 × (−320)
Answer: −160

7 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) lies on a rough table, $\mu = 0.4$, joined by a light string over a smooth pulley to particle B (4 kg), 1 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $4g - T = 4a$; A: $T - 0.4 \times 1g = 1a$
Adding: $(4 - 0.4 \times 1)g = 5a$, so $a = 7.056$
Speed of B and A as B lands: $v^2 = 2 \times 7.056 \times 1 = 14.112$
After B lands, A decelerates at $\mu g = 0.4 \times 9.8 = 3.92$
$0 = v^2 - 2(\mu g)s$, $s = 14.112 \div 7.84$

Answer: (a) 7.056 m s^{-2} (b) 1.8 m

8 Separable differential equations (All boards)

Solve the differential equation

$\frac{dy}{dx} = 3y \sin x$,

given that $y = 1$ when $x = \frac{\pi}{2}$. Give y in terms of x.

Separate: $\int \frac{1}{y}\, dy = \int 3\sin x\, dx$, so $\ln y = -3\cos x + C$

$x = \frac{\pi}{2}$, $y = 1$ gives $0 = 0 + C$, so $C = 0$

$y = e^{-3\cos x}$

Answer: $y = e^{-3\cos x}$

9 Exponential equations reducing to quadratics (All boards)

Solve $e^{2x} - 11e^x + 28 = 0$,
giving exact answers.

Let $u = e^x$: $u^2 - 11u + 28 = 0$, so $(u - 4)(u - 7) = 0$
$e^x = 4$ or $e^x = 7$

Answer: $x = 2 \ln 2$ or $x = \ln 7$

10 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 6 and its sum to infinity is 32.
Find the two possible values of the first term.

$ar = 6$ and $\frac{a}{(1 - r)} = 32$
$a = 32(1 - r)$, so $32(1 - r)\, r = 6$
$32r^2 - 32r + 6 = 0$: $r = \frac{1}{4}$ or $r = \frac{3}{4}$
$a = 32(1 - r) = 24$ or 8

Answer: $a = 8$ or $a = 24$