Day 62 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 80 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $80 = 7\times10 + \frac{1}{2}\times a\times10^2$, so $a = 0.2$
(b) $v = u + at = 7 + 0.2\times10 = 9$
(c) For the midpoint, $s = 40$: $v^2 = u^2 + 2as = 7^2 + 2\times0.2\times40$
$v^2 = 65$
Answer: (a) 0.2 m s^{-2} (b) 9 m s^{-1} (c) 8.06 m s^{-1}

2 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 2 + 3\cos\theta$, $y = 1 + 3\sin\theta$.
Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x-2}{3}$, $\sin\theta = \frac{y-1}{3}$
Use $\cos^2\theta + \sin^2\theta = 1$: $(x-2)^2 + (y-1)^2 = 9$
Answer: $(x-2)^2 + (y-1)^2 = 9$, a circle with centre (2, 1), radius 3

3 Poisson probabilities (CIE)

Flaws occur in a metre of cable at a mean rate of 3 per metre, as a Poisson process.
Find the probability that in one such period there are
(a) exactly 5 (b) at least 5 (c) fewer than 5 (3 s.f.).

$X \sim Po(3)$
(a) $P(X = 5) = e^{-3}\,3^5/120! = 0.101$
(b) $1 - P(X \leq 4) = 1 - 0.815 = 0.185$
(c) $P(X \leq 4) = 0.815$
Answer: (a) 0.101 (b) 0.185 (c) 0.815

4 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{1}{\sqrt{5}-1} + \frac{5}{\sqrt{5}+1}$ can be written as $p + q\sqrt{5}$, and find p and q.

Common denominator $(\sqrt{5}-1)(\sqrt{5}+1) = 5 - 1 = 4$
Numerator $= 1(\sqrt{5}+1) + 5(\sqrt{5}-1)$
$= -4 + 6\sqrt{5}$
Divide by 4.
Answer: $p = -1$, $q = \frac{3}{2}$

5 Exact values from the addition formulae (All boards)

By writing 165° as 120° + 45°, find the exact value of

cos 165°, in the form $\frac{\sqrt{a} \pm \sqrt{b}}{c}$.

$\cos 165° = \cos(120° + 45°) = \cos 120°\cos 45° - \sin 120°\sin 45°$
$= (-\frac{1}{2}) \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$
$= \frac{-\sqrt{2}-\sqrt{6}}{4}$
Answer: $\frac{-\sqrt{2}-\sqrt{6}}{4}$

6 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 + 5x - 10}{(x-1)(x-2)}$ in the form $1 + \frac{A}{(x-1)} + \frac{B}{(x-2)}$, where A and B are constants to be found.

Divide: $x^2 + 5x - 10 = (x^2 - 3x + 2) + (8x - 12)$

So the fraction is $1 + \frac{8x - 12}{(x-1)(x-2)}$

$A(x - 2) + B(x - 1) \equiv 8x - 12$

$x = 1$ gives $A = 4$; $x = 2$ gives $B = 4$

Answer: A = 4, B = 4

7 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 12 m s^{-1} from a point 44.1 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $44.1 = \frac{1}{2} \times 9.8 \times t^2$, so $t = 3$

(a) Horizontal distance = 12×3

(b) Vertical speed on landing = $9.8 \times 3 = 29.4$; horizontal speed = 12

Speed = $\sqrt{(12^2 + 29.4^2)} = 31.8$

$\tan \theta = 29.4 \div 12$, $\theta = 67.8°$

Answer: (a) 36 m (b) 31.8 m s^{-1} at 67.8° below the horizontal

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

lg y = lg A + n lg x: gradient n, intercept lg A

$n = \frac{9 - 6}{2 - 1} = 3$

At (1, 6): lg $A = 6 - 3 \times 1 = 3$, so $A = 10^3 = 1000$

Answer: n = 3, A = 1000

9 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

A circular ripple has area A cm^2 increasing at a constant 8 $cm^2 s^{-1}$.
Find the rate at which the circumference is increasing when the radius is 10 cm.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$ and $\frac{dr}{dt} = \frac{8}{2\pi r}$

$C = 2\pi r$, so $\frac{dC}{dt} = 2\pi \times \frac{dr}{dt} = 2\pi \times \frac{8}{2\pi \times 10} = \frac{4}{5}$

Answer: $\frac{4}{5}$ cm/s

10 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The masses in grams of two groups: factory A has n = 10, mean 26, sd 4;
factory B has n = 10, mean 40, sd 2.
Find the mean and standard deviation of all 20 values (3 s.f. for the sd).

$\Sigma x = 10 \times 26 + 10 \times 40 = 660$, mean = 33

Σx^2 (A) $= 10(4^2 + 26^2) = 6920$

Σx^2 (B) $= 10(2^2 + 40^2) = 16040$

Variance = $\frac{22960}{20} - 33^2 = 59$, so sd = 7.68

Answer: mean 33, sd 7.68