Day 61 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Composite functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x - 6$ and $g(x) = x^2 - 6$.
Solve $gf(x) = 3$.

$gf(x) = g(x - 6) = (x - 6)^2 - 6$
$(x - 6)^2 - 6 = 3$
$x - 6 = \pm 3$
Answer: x = 3 or x = 9

2 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 112 items.
The 4th item chosen is number 15 and the 6th item chosen is number 23.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(23 - 15) \div (6 - 4) = 4$
First item = $15 - 3 \times 4 = 3$
Sample size = $112 \div 4 = 28$
Answer: Interval 4, first 3, size 28

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^5 \left(\frac{3}{x} + \frac{1}{x^2}\right) dx$.

Integrate: $[3 \ln x - \frac{1}{x}]$

Substitute the limits and subtract: $\frac{15 \ln 5 + 4}{5}$

Answer: $\frac{15 \ln 5 + 4}{5}$

4 Transformations from a graph (All boards)

The diagram shows the curve $y = f(x)$, with turning points A and B.
Find the coordinates of the turning points of
(a) $y = 2f(x)$ (b) $y = f(x) + 2$

(a) $y = 2f(x)$: A → (1, −2), B → (3, −10)
(b) $y = f(x) + 2$: A → (1, 1), B → (3, −3)
Answer: (a) (1, −2), (3, −10); (b) (1, 1), (3, −3)

5 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take $g = 9.8$. Give answers to 3 s.f. if not exact.

$\sin \alpha = \frac{5}{13}$, $\cos \alpha = \frac{12}{13}$, $\sin \beta = 0.8$, $\cos \beta = 0.6$

Horizontal: $T_1 \cos \alpha = T_2 \cos \beta$, so $T_2 = \frac{20}{13} T_1$

Vertical: $T_1 \sin \alpha + T_2 \sin \beta = 4\ g = 39.2$

$T_1(\frac{5}{13} + \frac{16}{13}) = 39.2$

$T_1 = 24.3$, $T_2 = 37.3$

Answer: T_1 = 24.3 N, T_2 = 37.3 N

6 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{3}{5}$ and $\cos B = -\frac{4}{5}$.

Find the exact values of tan(A−B) and tan(A+B).

$\cos A = \frac{4}{5}$ (A acute), $\sin B = \frac{3}{5}$ (B obtuse), so $\tan A = \frac{3}{4}$, $\tan B = -\frac{3}{4}$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{3}{4} - (-\frac{3}{4})}{1 + \frac{3}{4} \times (-\frac{3}{4})} = \frac{24}{7}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + (-\frac{3}{4})}{1 - \frac{3}{4} \times (-\frac{3}{4})} = 0$$

Answer: $\tan(A-B) = \frac{24}{7}$, $\tan(A+B) = 0$

7 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 14 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction is 0.25. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

Up: deceleration = $g(\sin\alpha + \mu\cos\alpha) = 9.8(0.6 + 0.25\times0.8) = 7.84$

(a) $0 = 14^2 - 2\times7.84\times d$, so $d = 12.5$

Down: $\tan\alpha = \frac{3}{4} > \mu$, so it slides back; acceleration $= g(\sin\alpha - \mu\cos\alpha) = 3.92$

(b) $v^2 = 2\times3.92\times12.5 = 98$

$v = \sqrt{98}$

Answer: (a) 12.5 m (b) 9.90 m s^{-1}

8 Range of a composite function (All boards)

$f(x) = x^2 - 2x + 3$, $x \in \mathbb{R}$, and $g(x) = \frac{10}{x+2}$, $x > -2$.

(a) Find the range of f.

(b) Show that gf is defined and find its range.

$f(x) = (x-1)^2 + 2 \geq 2$

So $f(x) \geq 2 > -2$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{10}{f(x)+2}$, with $f(x) \geq 2$, so $f(x) + 2 \geq 4$

Largest value is $\frac{10}{4} = \frac{5}{2}$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 2$ (b) $0 < gf(x) \leq \frac{5}{2}$

9 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 37 students in sets A and B.

(a) Find the value of x.

(b) A student in B is chosen at random. Find P(A).

(a) Total: $2x + x + (x+1) + 4 = 37$

$4x + 5 = 37$, so $x = 8$

(b) Regions: 16, 8, 9, 4

In B: 8 + 9 = 17, of which in A: 8

Answer: (a) $x = 8$ (b) $\frac{8}{17}$

10 Fixed point iteration (All boards)

The equation $e^x = 6x + 5$ has a root α. Use the iteration $x_{n+1} = \ln(6x + 5)$ with $x_0 = 3$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \ln(6 \times 3 + 5) = 3.1355$

$x_2 = 3.1702$, $x_3 = 3.1789$

Continuing, the values settle at 3.1818, so $\alpha = 3.18$

Answer: $x_1 = 3.1355$, $x_2 = 3.1702$, $x_3 = 3.1789$; $\alpha = 3.18$