Day 60 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$.

Integrate: $[\frac{1}{2} \tan 2x]$

Substitute the limits and subtract: $\frac{1}{2}$

Answer: $\frac{1}{2}$

2 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = $-\frac{1}{8}$, so gradient of the perpendicular = 8

Through C: $8x - y = -23$

Line AB: $y - 4 = -\frac{1}{8}(x - -4)$. Solve to get D = $(-\frac{12}{5}, \frac{19}{5})$

Area = ½ × AB × CD = 13

Answer: (a) $8x - y = -23$ (b) D$(-\frac{12}{5}, \frac{19}{5})$, area 13

3 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a(\frac{8}{9})$ in terms of p and q.

$\frac{8}{9} = 2^3 \times 3^{-2}$

$\log_a(2^3 \times 3^{-2}) = 3 \log_a 2 - 2 \log_a 3$
$= 3p - 2q$
Answer: $3p - 2q$

4 Transformations from a graph (All boards)

The diagram shows the curve y = f(x), with turning points A and B.
Find the coordinates of the turning points of
(a) y = −f(x) (b) y = 3f(x)

(a) y = −f(x): A → (−3, −4), B → (−1, 4)
(b) y = 3f(x): A → (−3, 12), B → (−1, −12)
Answer: (a) (−3, −4), (−1, 4); (b) (−3, 12), (−1, −12)

5 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(3t - 6)$ m s^{-2}. When t = 0 it is at O with velocity 8 m s^{-1}.
Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 2
$v = \int a \, dt = 1.5t^2 - 6t + 8$ (using v = 8 when t = 0)
v(2) = 2
$x = \int v \, dt = 0.5t^3 - 3t^2 + 8t$ (x = 0 when t = 0)
x(2) = 8
Answer: (a) 2 m s^{-1} (b) 8 m

6 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{7} - 1) = 2 + \sqrt{7}$, writing x in the form $p + q\sqrt{7}$.

$x = \dfrac{2 + \sqrt{7}}{\sqrt{7} - 1}$

Multiply top and bottom by $\sqrt{7} + 1$.

Numerator: $(2 + \sqrt{7})(\sqrt{7} + 1) = 9 + 3\sqrt{7}$

Denominator: $7 - 1 = 6$

Answer: $\mathbf{x = \frac{3}{2} + \frac{1}{2}\sqrt{7}}$

7 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 - 4t)\mathbf{i} + (t^2 - 4t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when t = 4, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 - 4)\mathbf{i} + (2t - 4)\mathbf{j}$

(a) $t = 4$: $\mathbf{v} = 44\mathbf{i} + 4\mathbf{j}$

Speed $= \sqrt{(44^2 + 4^2)} = \sqrt{1952}$

(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $2t - 4 = 0$, so $t = 2$

Answer: (a) 44.2 m s^{-1} (b) t = 2

8 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is $N(171, 7^2)$. The lowest 95% of values are below k.
Find k to 3 significant figures.

$P(X < k) = 0.95$

$z = \Phi^{-1}(0.95) = 1.64$

$k = 171 + (1.64) \times 7 = 183$

Answer: 183

9 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates $(-1, 1, -2)$, $(-2, -2, 3)$ and $(-3, 2, -3)$.
Find angle ABC, in degrees to 1 d.p.

$\mathrm{BA} = \begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}$, $\mathrm{BC} = \begin{pmatrix} -1 \\ 4 \\ -6 \end{pmatrix}$

$\mathrm{BA} \cdot \mathrm{BC} = 41$

$|\mathrm{BA}| = \sqrt{35}$, $|\mathrm{BC}| = \sqrt{53}$

$\cos \mathrm{ABC} = \dfrac{41}{\sqrt{35} \times \sqrt{53}} = 0.9519$, so angle ABC = 17.8°

Answer: 17.8°

10 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mass of an egg is $N(41, 8^2)$. An egg is classed as 'special' if its mass is more than 56.
6 eggs are chosen at random. Find the probability that exactly 2 are special (3 s.f.).

$P(\text{special}) = P(X > 56) = P(Z > 1.88) = 0.0304$

$Y \sim B(6, 0.0304)$

$P(Y = 2) = C(6, 2)(0.0304)^2(0.970)^4 = 0.0122$

Answer: 0.0122