Day 6 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{4}{5}$.

Find the exact values of $\sin 2\theta$, $\cos 2\theta$ and $\tan 2\theta$.

$\cos\theta = -\sqrt{1 - \frac{4}{5}^2} = -\frac{3}{5}$ (obtuse)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{4}{5} \times (-\frac{3}{5}) = -\frac{24}{25}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \frac{4}{5}^2 = -\frac{7}{25}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{24}{7}$

Answer: $\sin 2\theta = -\frac{24}{25}$, $\cos 2\theta = -\frac{7}{25}$, $\tan 2\theta = \frac{24}{7}$

2 Arithmetic sequences: finding a and d (All boards)

The 6th term of an arithmetic sequence is 13 and the 13th term is 27.

Find the first term and common difference, and the sum of the first 30 terms.

$u_{13} - u_6 = 7\,d = 27 - 13$ so $d = 2$

$a + 5\,d = 13$ so $a = 3$

$S_{30} = \frac{30}{2}(2 \times 3 + 29 \times 2)$

Answer: $a = 3$, $d = 2$, $S_{30} = 960$

3 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_1^5 5(\ln x)\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 1, 2, 3, 4, 5: 0.0000, 3.4657, 5.4931, 6.9315, 8.0472

$T = \frac{1}{2}[0.0000 + 8.0472 + 2(3.4657 + 5.4931 + 6.9315)]$

T = 19.9

The curve is concave ($y'' < 0$) on this interval, so the chords lie below the curve: underestimate

Answer: 19.9, an underestimate

4 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 14.7 m s^{-1} from a point h m above horizontal ground, and hits the ground 7 s later.

Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 14.7$, $a = -9.8$, $t = 7$

$-h = 14.7\times7 - 4.9\times7^2$, so $h = 137.2$

(b) At the highest point $v = 0$: $0 = 14.7^2 - 2\times9.8\times s$

s = 11.025 m above the point of projection

Greatest height above the ground = 137.2 + 11.025

Answer: (a) 137.2 m (b) 148.225 m

5 Factor theorem: roots with surds (All boards)

$f(x) = x^3 + 3x^2 - 3x - 1$.

(a) Show that $(x - 1)$ is a factor of f(x).

(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(1) = 1 + 3 - 3 - 1 = 0$, so $(x - 1)$ is a factor.

$f(x) = (x - 1)(x^2 + 4x + 1)$

$x^2 + 4x + 1 = 0$ gives $x = \frac{-4 \pm \sqrt{12}}{2}$

$\sqrt{12} = 2\sqrt{3}$

Answer: $x = 1$ or $x = -2 \pm \sqrt{3}$

6 Three-set Venn diagram counts (All boards)

Of 33 students, 13 study Maths (M), 16 Physics (P) and 15 Chemistry (C).
7 study M and P, 5 study M and C, 8 study P and C, and 3 study all three.
One student is chosen at random. Given that a student studies Chemistry, find the probability that they study all three.

Venn regions: M only 4, P only 4, C only 5,
MP only 4, MC only 2, PC only 5, all three 3, outside all 6
Studies Chemistry: 15
Studies all three: 3
Probability = $\frac{3}{15}$

Answer: $\frac{1}{5}$

7 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 112.8 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $112.8 = 7\times12 + \frac{1}{2}\times a\times12^2$, so $a = 0.4$
(b) $v = u + at = 7 + 0.4\times12 = 11.8$
(c) For the midpoint, $s = 56.4$: $v^2 = u^2 + 2as = 7^2 + 2\times0.4\times56.4$
$v^2 = 94.12$

Answer: (a) 0.4 m s^{-2} (b) 11.8 m s^{-1} (c) 9.70 m s^{-1}

8 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = -3 + 4\cos\theta$, $y = 1 + 4\sin\theta$.
Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x+3}{4}$, $\sin\theta = \frac{y-1}{4}$

Use $\cos^2\theta + \sin^2\theta = 1$: $(x+3)^2 + (y-1)^2 = 16$

Answer: $(x+3)^2 + (y-1)^2 = 16$, a circle with centre $(-3, 1)$, radius 4

9 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^2 x(x^2+1)^4\,dx$.

$du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$; $x = 0$ gives $u = 1$, $x = 2$ gives $u = 5$

$I = \frac{1}{2}\int_1^5 u^4\,du = \frac{1}{2}\left[\frac{u^5}{5}\right]$
$= \frac{1562}{5}$

Answer: $\frac{1562}{5}$

10 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function $f(x) = kx(4-x)$ for $0 \le x \le 4$, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find $P(X > \frac{4}{3})$.

(a) $\int_0^4 f(x)\,dx = 1$ gives $\frac{32}{3}k = 1$, so $k = \frac{3}{32}$
(b) $E(X) = \int x\,f(x)\,dx = 2$
(c) $P(X > \frac{4}{3}) = \int_{\frac{4}{3}}^4 f(x)\,dx = \frac{20}{27}$

Answer: (a) $k = \frac{3}{32}$ (b) 2 (c) $\frac{20}{27}$