Day 59 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Algebraic fractions (All boards)

Simplify fully $\frac{9x^2 - 25}{3x^2 - 17x + 20}$.

Factorise the numerator: $(3x - 5)(3x + 5)$
Factorise the denominator: $(3x - 5)(x - 4)$
Cancel the common factor $(3x - 5)$.

Answer: $\mathbf{\frac{3x + 5}{x - 4}}$

2 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 5$, $|b| = 2$ and the angle between a and b is 150°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 150° = 5 \times 2 \times -\frac{\sqrt{3}}{2} = -5\sqrt{3}$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 25 + -10\sqrt{3} + 4 = 29 - 10\sqrt{3}$

$|a + b| = \sqrt{29 - 10\sqrt{3}}$

Answer: $\mathbf{a \cdot b = -5\sqrt{3}}$**,** $\mathbf{|a + b| = \sqrt{29 - 10\sqrt{3}}}$

3 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−13, −8) and B(11, 2) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(4, −15) lies on the circle.

Centre = midpoint of AB = (−1, −3)

$r^2 = (-12)^2 + (-5)^2 = 169$

$(x + 1)^2 + (y + 3)^2 = 169$: $x^2 + y^2 + 2x + 6y - 159 = 0$

(b) $CM^2 = (5)^2 + (-12)^2 = 169 = r^2$

Answer: (a) $\mathbf{x^2 + y^2 + 2x + 6y - 159 = 0}$

4 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 3120 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(150 + 110) × V
3120 = 130 V, so V = 24
(b) Deceleration = V ÷ 20 = 24 ÷ 20

Answer: (a) $\mathbf{V = 24}$ **(b)** $\mathbf{1.2\ m\ s^{-2}}$

5 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve C: $y = x(9 - x)$ and the line L: $y = 7x$.
Find the area of the shaded region between C and L.

Intersect: $x(9 - x) = 7x$ gives $x = 0$ or $x = 2$

Area $= \int (C - L)\,dx = \int_0^2 (2x - x^2)\,dx = [\frac{2x^2}{2} - \frac{x^3}{3}]$

$= \frac{2^3}{2} - \frac{2^3}{3} = \frac{4}{3}$

Answer: $\mathbf{\frac{4}{3}}$

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6 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

20 values have $\Sigma x = 820$ and $\Sigma x^2 = 33800$.
One value was recorded as 45 but should have been 44.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 820 - 45 + 44 = 819$

$\Sigma x^2 = 33800 - 45^2 + 44^2 = 33711$

Mean $= \frac{819}{20} = 41.0$

sd $= \sqrt{\frac{33711}{20} - (41.0)^2} = 2.94$

Answer: mean 41.0, sd 2.94

7 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + 2y^2 = 6$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 4y}$

Horizontal tangent: $4x + 2y = 0$, so $y = -2x$

Substitute: $6x^2 = 6$, so $x^2 = 1$ and $x = \pm 1$

Points: $(1, -2)$ and $(-1, 2)$

Answer: $(1, -2)$ and $(-1, 2)$

8 Inverse of a rational function (All boards)

$f(x) = \frac{2x - 3}{x - 5}$, $x \in \mathbb{R}$, $x \neq 5$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{2x - 3}{x - 5}$: $y(x - 5) = 2x - 3$

$xy - 5y = 2x - 3$

$x(y - 2) = 5y - 3$

$x = \frac{5y - 3}{y - 2}$, then swap x and y

Answer: $f^{-1}(x) = \frac{5x - 3}{x - 2}$, $x \neq 2$

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(25, p)$. H_0: $p = 0.4$ is tested against H_1: $p > 0.4$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.5$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 15$

(b) Type I error = P(reject H_0 | $p = 0.4$) = 0.0344

(c) Type II error = P(not in critical region | $p = 0.5$)

$P(X \leq 14)$ when $p = 0.5$: 0.788

Answer: (a) $X \geq 15$ (b) 0.0344 (c) 0.788

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (10 m, 30 kg) rests with A on rough ground, $\mu = 0.3$, and B against a smooth wall, B being 8 m above the ground. A man of mass 75 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(10^2 - 8^2)} = 6$ m from the wall

Vertical: $R = (30 + 75)g$. Limiting friction: $F = \mu R = 0.3(30 + 75)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 6/10$):

$S \times 8 = 30g \times 3 + 75g \times (6x \div 10)$

$31.5 \times 8 = 90 + 45x$ (g cancels), so $x = 3.6$

Answer: 3.6 m