**Day 57** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle rests on a rough plane inclined at $\alpha$, $\tan\alpha = \frac{12}{5}$, $\mu = 0.5$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, $R = 4g\cos\alpha = \frac{196}{13}$ N

Limiting friction $= \mu R = \frac{98}{13}$ N

P smallest: friction acts up the slope: $P + \mu R = 4g\sin\alpha$, $P = \frac{2352}{65} - \frac{98}{13}$

P greatest: friction acts down the slope: $P = \frac{2352}{65} + \frac{98}{13}$

**Answer: $28.6\text{ N} \leq P \leq 43.7\text{ N}$**

## 2 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 6\sqrt{x} + 5 = 0$.

Let $u = \sqrt{x}$: $u^2 - 6u + 5 = 0$
$(u - 5)(u - 1) = 0$ so $u = 5$ or $1$
$\sqrt{x} = 5$ gives $x = 25$; $\sqrt{x} = 1$ gives $x = 1$
**Answer: $x = 1$ or $x = 25$**

## 3 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A club has 180 members: 100 Bronze, 30 Silver and 50 Gold.
A stratified sample of size 18 is taken.
Find the number of members sampled from each group.

Sampling fraction $= \frac{18}{180} = \frac{1}{10}$

Bronze: $100 \times \frac{1}{10} = 10$

Silver: $30 \times \frac{1}{10} = 3$

Gold: $50 \times \frac{1}{10} = 5$

**Answer: Bronze 10, Silver 3, Gold 5**

## 4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x-3} = 9^{3x+2}$.

Write both sides as powers of 3.
$(3^1)^{2x-3} = (3^2)^{3x+2}$
$2x - 3 = 2(3x + 2)$
$2x - 3 = 6x + 4$
$-4x = 7$
**Answer: $x = -\frac{7}{4}$**

## 5 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(2\arccos\frac{3}{5})$.

Let $\theta = \arccos\frac{3}{5}$, so $\cos\theta = \frac{3}{5}$ with $\theta$ acute

$\sin\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$

**Answer: $\frac{24}{25}$**

## 6 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 35 students in sets A and B.

(a) Find the value of x.

(b) A student is chosen at random. Find P(A′ | B′).

(a) Total: 2x + (x + 2) + 3x + 3 = 35

6x + 5 = 35, so x = 5

(b) Regions: 10, 7, 15, 3

Not in B: 13, of which not in A: 3

**Answer: (a) x = 5 (b) $\frac{3}{13}$**

## 7 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(3, −1, 0) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} -1 \\ 3t - 3 \\ -2 \end{pmatrix}$

PF·d = 0 gives 9t − 9 = 0, so t = 1

F = (2, −1, −2); $\mathbf{PF} = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$, so |PF| = $\sqrt{5}$

**Answer: F(2, −1, −2), distance $\sqrt{5}$**

## 8 Periodic sequences (All boards)

A sequence is defined by $u_1 = 5$ and $u_{n+1} = 1 - \frac{1}{u_n}$.

(a) Find $u_2$ and $u_3$.

(b) Find $u_{99}$ and $\sum_{r=1}^{99} u_r$.

$u_2 = \frac{4}{5}$, $u_3 = -\frac{1}{4}$, $u_4 = 5$

$u_4 = u_1$, so the sequence repeats with period 3.

99 = 3 × 33 + 0, so $u_{99} = u_3 = -\frac{1}{4}$

Sum of one period = $\frac{111}{20}$; 33 full periods plus 0 extra term(s)

**Answer: (a) $\frac{4}{5}$, $-\frac{1}{4}$ (b) $u_{99} = -\frac{1}{4}$, sum = $\frac{3663}{20}$**

## 9 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 99 m ahead. After a reaction time of 1 s she brakes with deceleration 2 m $s^{-2}$ and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 1 v, braking distance = $v^2 \div (2\times2)$

$1\,v + v^2 \div 4 = 99$, so $v^2 + 4\,v - 396 = 0$

(v − 18)(v + 22) = 0, so v = 18

(b) Braking time = 18 ÷ 2 = 9 s; total = 1 + 9

**Answer: (a) 18 m $s^{-1}$ (b) 10 s**

## 10 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 3, |b| = 8 and the angle between a and b is 120°.
Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 120° = 3 \times 8 \times -\frac{1}{2} = -12$

$|\mathbf{a} + \mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 9 + -24 + 64 = 49$
$|\mathbf{a} + \mathbf{b}| = 7$
**Answer: $\mathbf{a}\cdot\mathbf{b} = -12$, $|\mathbf{a} + \mathbf{b}| = 7$**