Day 56 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = \cos^3(2x)$. Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{8}$.

Let $u = \cos(2x)$: $y = u^3$, so $\frac{dy}{dx} = 3u^2 \times \frac{du}{dx}$

$\frac{du}{dx} = -2\sin(2x)$, giving $\frac{dy}{dx} = -6\cos^2(2x)\sin(2x)$

At $x = \frac{\pi}{8}$: $\cos(2x) = \frac{\sqrt{2}}{2}$ and $\sin(2x) = \frac{\sqrt{2}}{2}$, so gradient $= -\frac{3\sqrt{2}}{2}$

Answer: $\frac{dy}{dx} = -6\cos^2(2x)\sin(2x)$; gradient $= -\frac{3\sqrt{2}}{2}$

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 24.5 m s^{-1} from a point h m above horizontal ground, and hits the ground 8 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 24.5$, $a = -9.8$, $t = 8$
$-h = 24.5\times8 - 4.9\times8^2$, so $h = 117.6$
(b) At the highest point $v = 0$: $0 = 24.5^2 - 2\times9.8\times s$
$s = 30.625$ m above the point of projection
Greatest height above the ground = 117.6 + 30.625
Answer: (a) 117.6 m (b) 148.225 m

3 Double angle equations (All boards)

Solve $\cos 2\theta + 2\sin\theta = 1$
for $0 \leq \theta \leq 2\pi$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta - 2\sin\theta = 0$
Solve the quadratic: $\sin\theta = 1$ or $\sin\theta = 0$

$\sin\theta = 1$: $\theta = \frac{\pi}{2}$

$\sin\theta = 0$: $\theta = 0, \pi, 2\pi$

Answer: $\theta = 0, \frac{\pi}{2}, \pi, 2\pi$

4 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $\frac{2}{x^2} - \frac{14}{x} + 12 = 0$.

Let $u = \frac{1}{x}$: $2u^2 - 14u + 12 = 0$

$u = 1$ or $u = 6$

$x = \frac{1}{u}$: $x = 1$ or $x = \frac{1}{6}$

Answer: $x = \frac{1}{6}$ or $x = 1$

5 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(27, 0.5)$.
Find the smallest value of c such that $P(X \geq c) < 0.1$.

$P(X \geq 18) = 1 - P(X \leq 17) = 0.0610$
$P(X \geq 17) = 1 - P(X \leq 16) = 0.124$
$0.0610 < 0.1$ but $0.124 > 0.1$
Answer: c = 18

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6 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that $\frac{1-\cos 5\theta}{\theta \sin \theta}$ approaches as $\theta \to 0$.

$1-\cos 5\theta \approx \frac{1}{2}(5\theta)^2 = \frac{25}{2}\theta^2$

$\sin \theta \approx \theta$

Expression $\approx \frac{\frac{25}{2}\theta^2}{\theta \times \theta} = \frac{25}{2} = \frac{25}{2}$

Answer: $\frac{25}{2}$

7 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 900 kg car tows a 400 kg trailer by a light tow-bar on a horizontal road. Driving force 1240 N; resistances 600 N (car), 250 N (trailer). Find (a) the acceleration and the tow-bar tension. At 10 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1240 - 600 - 250 = 1300\,a$, so $a = 0.3$

Trailer: $T - 250 = 400\,a$, so $T = 250 + 400\times 0.3 = 370$

(b) After the break the trailer decelerates at $250 \div 400 = 0.625$ m s^{-2}

$0 = 10^2 - 2\times 0.625\times s$, so $s = 80$

Answer: (a) 0.3 m s^{-2}, 370 N (b) 80 m

8 Asymptotes after transformation (All boards)

The curve $y = f(x)$, where $f(x) = \frac{1}{x-2} - 1$, is translated by $\begin{pmatrix}-3\\-3\end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of $y = f(x)$: $x = 2$, $y = -1$

Translate by $\begin{pmatrix}-3\\-3\end{pmatrix}$: $x = -1$, $y = -4$

New curve: $y = \frac{1}{x+1} - 4$

$y = 0$: $\frac{1}{x+1} = 4$, so $x + 1 = \frac{1}{4}$

Answer: $x = -1$, $y = -4$; crosses x-axis at $(-\frac{3}{4}, 0)$

9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1-x)^{\frac{1}{2}}$ in ascending powers of x up to x^2.

(b) Use $x = 0.01$ to estimate $\sqrt{99}$ to 4 decimal places.

$(1+y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -1x$

(a) $1 - \frac{1}{2}x - \frac{1}{8}x^2$

$x = \frac{1}{100}$: $99 = 10^2(1 - \frac{1}{100})$ so $\sqrt{99} = 10 \times (1 - \frac{1}{100})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{200} - \frac{1}{80000}) = 9.949875$

Answer: (a) $1 - \frac{1}{2}x - \frac{1}{8}x^2$ (b) 9.9499

10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(34, p)$. H_0: $p = 0.5$ is tested against H_1: $p < 0.5$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.4$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 11$

(b) Type I error = P(reject H_0 | $p = 0.5$) = 0.0288

(c) Type II error = P(not in critical region | $p = 0.4$)

$P(X \geq 12)$ when $p = 0.4$: 0.767

Answer: (a) $X \leq 11$ (b) 0.0288 (c) 0.767