Day 55 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 8 m s^{-1} and accelerates uniformly. She travels 83.2 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $83.2 = 8\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.6$
(b) $v = u + at = 8 + 0.6\times8 = 12.8$
(c) For the midpoint, $s = 41.6$: $v^2 = u^2 + 2as = 8^2 + 2\times0.6\times41.6$
$v^2 = 113.92$

Answer: (a) 0.6 m s^{-2} (b) 12.8 m s^{-1} (c) 10.7 m s^{-1}

2 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.
Find the exact values of A and k.

ln y = ln A + kx, a straight line with gradient k and intercept ln A
Intercept ln A = 3, so $A = e^3$
Gradient $k = \frac{9-3}{4-0} = \frac{3}{2}$

Answer: $A = e^3$, $k = \frac{3}{2}$

3 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.
Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point $(-2, 7)$: $f(x) = a(x + 2)^2 + 7$
$f(0) = 11$: $a(-2)^2 + 7 = 11$
$a = 1$, then expand

Answer: $f(x) = x^2 + 4x + 11$

4 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{15}{17}$ and $\cos B = -\frac{4}{5}$.
Find the exact values of sin(A−B) and cos(A+B).

cos A = $\frac{8}{17}$ (A acute), sin B = $\frac{3}{5}$ (B obtuse), so tan A = $\frac{15}{8}$, tan B = $-\frac{3}{4}$

$\sin(A-B) = \sin A\cos B - \cos A\sin B = \frac{15}{17}\times(-\frac{4}{5}) - \frac{8}{17}\times\frac{3}{5} = -\frac{84}{85}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B = \frac{8}{17}\times(-\frac{4}{5}) - \frac{15}{17}\times\frac{3}{5} = -\frac{77}{85}$

Answer: $\sin(A-B) = -\frac{84}{85}$, $\cos(A+B) = -\frac{77}{85}$

5 Normal approximation to the binomial (All boards)

X ~ B(40, 0.4). Use a normal approximation to estimate $P(X \geq 18)$ to 3 significant figures.

np = 16, npq = 9.6, so Y ~ N(16, 9.6)
With continuity correction: P(Y > 17.5)
$z = \frac{17.5 - 16}{3.10} = 0.484$, P(Z > 0.484) = 0.314

Answer: 0.314

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6 Range of a composite function (All boards)

$f(x) = x^2 - 2x + 7$, $x \in \mathbb{R}$, and $g(x) = \frac{3}{x-4}$, $x > 4$.

(a) Find the range of f.

(b) Show that gf is defined and find its range.

$f(x) = (x-1)^2 + 6 \geq 6$

So $f(x) \geq 6 > 4$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{3}{f(x)-4}$, with $f(x) \geq 6$, so $f(x) - 4 \geq 2$

Largest value is $\frac{3}{2} = \frac{3}{2}$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 6$ (b) $0 < gf(x) \leq \frac{3}{2}$

7 Partial fractions and integration (All boards)

(a) Express $\frac{-x}{(x+4)(x+2)}$ in partial fractions.

(b) Hence find $\int_0^5 \frac{-x}{(x+4)(x+2)}\,dx$, giving the answer in the form ln k.

$-x \equiv A(x+2) + B(x+4)$: $x = -2$ gives $A = -2$; $x = -4$ gives $B = 1$

$I = [-2\ln(x+4) + \ln(x+2)]_0^5$

$= -2\ln\frac{9}{4} + \ln\frac{7}{2}$

$= \ln\frac{56}{81}$

Answer: (a) $\frac{-2}{x+4} + \frac{1}{x+2}$; (b) $\ln\frac{56}{81}$

8 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (6 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 5 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{3}{4}$. Find (a) the tension, (b) the magnitude of the hinge force. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 4 = 6g \times 2 + 5g \times 4$

$T\sin\theta = (3+5)g = 78.4$, so (a) $T = 131$

Horizontal component of hinge force: $X = T\cos\theta = 105$

Vertical: $Y + T\sin\theta = (6+5)g$, so $Y = 6g \div 2 = 29.4$

(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(105^2 + 29.4^2)}$

Answer: (a) 131 N (b) 109 N

9 Hypothesis test for a Poisson mean (CIE)

Faults occur as a Poisson process at a mean rate of 3 per roll of cloth.

After a change, the number recorded in one roll of cloth is 2.

Test at the 5% level whether the mean rate has decreased.

$H_0: \lambda = 3$, $H_1: \lambda < 3$; $X \sim Po(3)$

$P(X \leq 2) = 0.423$

$0.423 > 0.05$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.423; do not reject H_0

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{6x - 1}{(1 + 2x)(1 - 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 2x)} + \frac{B}{(1 - 2x)}$: $A = -2$, $B = 1$

(b) $-2(1 + 2x)^{-1} = -2(1 - 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|2x| < 1$

Answer: (a) $\frac{-2}{(1 + 2x)} + \frac{1}{(1 - 2x)}$ **(b)** $-1 + 6x - 4x^2$ **(c)** $|x| < \frac{1}{2}$