Day 54 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2x = -\sqrt{3}\cos x$ for $0° \leq x \leq 360°$.

$\sin 2x \equiv 2\sin x \cos x$: $\cos x (2\sin x + \sqrt{3}) = 0$
$\cos x = 0$: $x = 90°, 270°$

$\sin x = \frac{-\sqrt{3}}{2}$: $x = 240°, 300°$

Do not divide by cos x, as that would lose solutions
Answer: x = 90°, 240°, 270°, 300°

2 Sigma notation (All boards)

Evaluate $\sum_{r=5}^{24}(4r - 3)$.

Arithmetic series: first term 17, last term 93, 20 terms

$S = \frac{20}{2}(17 + 93) = 1100$

Answer: 1100

3 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_1^5 3(\sqrt{x})\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 1, 2, 3, 4, 5: 3.0000, 4.2426, 5.1962, 6.0000, 6.7082

$T = \frac{1}{2}[3.0000 + 6.7082 + 2(4.2426 + 5.1962 + 6.0000)]$

T = 20.3
The curve is concave (y'' < 0) on this interval, so the chords lie below the curve: underestimate
Answer: 20.3, an underestimate

4 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 10 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m. A 4 kg particle is placed at E, AE = x m, and the plank is about to tilt about D. Find x.

About to tilt about D, so the reaction at C is zero and E lies between D and B.
Moments about D: $4\,g \times (x - 2.5) = 10\,g \times (2.5 - 2)$
$4(x - 2.5) = 5$
$x = 2.5 + 1.25$
Answer: x = 3.75

5 Partial fractions and integration (All boards)

(a) Express $\frac{x+2}{(x+1)^2}$ in the form $\frac{A}{x+1} + \frac{B}{(x+1)^2}$.

(b) Hence find the exact value of $\int_0^3 \frac{x+2}{(x+1)^2}\,dx$.

$x + 2 \equiv A(x + 1) + B$, so A = 1 and B = 1

$I = [1\ln(x+1) - \frac{1}{x+1}]_0^3$

$= 1\ln\frac{4}{1} + 1(1 - \frac{1}{4})$

$= \frac{8\ln 2 + 3}{4}$

Answer: (a) A = 1, B = 1; (b) $\frac{8\ln 2 + 3}{4}$

6 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x + 3y = k$ is a tangent to the circle
$(x - 2)^2 + (y - 1)^2 = 25$.
Find the two possible values of k.

Centre (2, 1), radius 5
Distance from centre to the line = radius:
$\frac{|11 - k|}{5} = 5$
$|11 - k| = 25$
Answer: $k = -14$ or $k = 36$

7 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 8, mean 40, sd 9;
group Y has n = 16, mean 22, sd 4.
Find the mean and standard deviation of all 24 values (3 s.f. for the sd).

$\Sigma x = 8 \times 40 + 16 \times 22 = 672$, mean = 28
Σx^2 (A) $= 8(9^2 + 40^2) = 13448$
Σx^2 (B) $= 16(4^2 + 22^2) = 8000$
Variance $= \frac{21448}{24} - 28^2 = 109.666667$, so sd = 10.5
Answer: mean 28, sd 10.5

8 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, $\sin\beta = 0.6$, $\cos\beta = 0.8$
Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = 0.75\, T_1$
Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 2g = 19.6$
$T_1(0.8 + 0.45) = 19.6$
$T_1 = 15.7$, $T_2 = 11.8$
Answer: $T_1 = 15.68$ N, $T_2 = 11.76$ N

9 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line $y = 20 + 1.2x$.
(a) Interpret the gradient.
(b) Estimate y when x = 15 and comment.

(a) On average, y increases by 1.2 hundred sales for each extra 1 °C of x
(b) $y = 20 + 1.2 \times 15 = 38$
x = 15 is within the range of the data (6 to 16): interpolation, so reliable
Answer: (a) y increases by 1.2 hundred sales per °C (b) 38, reliable

10 Asymptotes after transformation (All boards)

The curve $y = f(x)$, where $f(x) = \frac{3}{x - 3} - 3$, is translated by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.
Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of $y = f(x)$: $x = 3$, $y = -3$
Translate by $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$: $x = 6$, $y = -2$
New curve: $y = \frac{3}{x - 6} - 2$
$y = 0$: $\frac{3}{x - 6} = 2$, so $x - 6 = \frac{3}{2}$
Answer: $x = 6$, $y = -2$; crosses x-axis at $(\frac{15}{2}, 0)$