Day 53 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Perth: mean 14.5, standard deviation 4.7;
Jacksonville: mean 8.5, standard deviation 2.4.
Make two comparisons.

Means: Perth has the higher mean (14.5 against 8.5), so on average the total rainfall is greater at Perth
Standard deviations: Perth is larger at Perth (4.7 against 2.4)
so the readings at Perth are more variable and those at Jacksonville are more consistent
Answer: Perth higher on average; Perth more variable

2 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: $x = 1$
Line AB: $y - 2 = 0(x - -3)$. Solve to get D = (1, 2)
Area = ½ × AB × CD = 18
Answer: (a) x = 1 (b) D(1, 2), area 18

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{12}} 5\sec^2 3x \, dx$.

Integrate: $[\frac{5}{3} \tan 3x]$

Substitute the limits and subtract: $\frac{5}{3}$

Answer: $\frac{5}{3}$

4 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve y = f(x), where f(x) is a cubic.
Find f(x), giving your answer in the form $k(x - p)(x - q)^2$.

Crosses at $x = -1$ and touches at $x = 1$, so $f(x) = k(x + 1)(x - 1)^2$
$f(0) = -2$: $k(1)(-1)^2 = -2$
$k = -2$
Answer: $\mathbf{f(x) = -2(x + 1)(x - 1)^2}$

5 Normal approximation to the Poisson (CIE)

X ~ Po(20). Use a suitable approximation to find $P(X \leq 24)$, to 3 significant figures.

$\lambda > 15$, so Y ~ N(20, 20) approximately
Continuity correction: P(Y < 24.5)
z = 1.01, so probability = 0.843
Answer: 0.843

6 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

lg y = lg A + n lg x: gradient n, intercept lg A
$n = \frac{-1 - 1}{2 - 1} = -2$

At (1, 1): lg A = 1 − (−2) × 1 = 3, so $A = 10^3 = 1000$
Answer: n = −2, A = 1000

7 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 40 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{5}{12}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component = $40\times\frac{12}{13} = \frac{480}{13}$; vertical component = $40\times\frac{5}{13} = \frac{200}{13}$

(a) Vertical: $0 = \frac{200}{13}t - 4.9t^2$, so $t = \frac{2000}{637}$

(b) At the top: $0 = \left(\frac{200}{13}\right)^2 - 2\times9.8\times H$, so $H = \frac{100000}{8281}$

(c) OA = $\frac{480}{13}\times\frac{2000}{637}$

Answer: (a) 3.14 s (b) 12.1 m (c) 116 m

8 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point (4, 2). Its centre lies in the first quadrant. Find the equations of the two possible circles.

Centre (r, r), radius r > 0.

Through (4, 2): $(4-r)^2 + (2-r)^2 = r^2$

$r^2 - 12r + 20 = 0$

$(r-2)(r-10) = 0$, so r = 2 or 10

Answer: $(x-2)^2 + (y-2)^2 = 4$ or $(x-10)^2 + (y-10)^2 = 100$

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}6\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}3\\-1\\-2\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}2\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}-2\\2\\3\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $6 + 3\lambda = 2 - 2\mu$; $1 - 1\lambda = 2 + 2\mu$; $-2 - 2\lambda = -1 + 3\mu$

Solving two of the equations gives a unique (λ, μ)

which does not satisfy the third, so the lines do not meet

Answer: The lines do not intersect (skew)

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (15 m, 25 kg) rests with A on rough ground, $\mu = 0.25$, and B against a smooth wall, B being 12 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(15^2 - 12^2)} = 9$ m from the wall

Vertical: R = (25 + 50)g. Limiting friction: F = μR = 0.25(25 + 50)g

Horizontal: S = F (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance x×9/15):

S×12 = 25 g×4.5 + 50 g×(9 x ÷ 15)

18.75×12 = 112.5 + 30 x (g cancels), so x = 3.75

Answer: 3.75 m