Day 52 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 40 members is taken from three groups.
6 of the sample are from Bronze, which has 48 members.
(a) Find the total number of members.
(b) Silver has 112 members. How many are sampled?

Bronze sampled fraction = $\frac{6}{48} = \frac{1}{8}$

Total = $40 \div \frac{1}{8} = 320$

Silver: $112 \times \frac{1}{8} = 14$

Answer: (a) 320 (b) 14

2 Logarithmic equations (All boards)

Solve $\log_2(x + 7) + \log_2(x - 7) = 5$.

Combine: $\log_2[(x + 7)(x - 7)] = 5$, so $(x + 7)(x - 7) = 2^5 = 32$
$x^2 - 81 = 0$: $(x - 9)(x + 9) = 0$
$x = 9$ or $x = -9$; $x = -9$ makes $x - 7$ negative, so it is rejected
Answer: $x = 9$

3 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(17, 0.25). Find, to 3 significant figures:
(a) P(X = 7) (b) P(X ≥ 7) (c) P(X ≤ 6)

(a) $C(17, 7)(0.25)^7(0.75)^{10} = 0.0668$
(b) $1 - P(X \leq 6) = 1 - 0.893 = 0.107$
(c) $P(X \leq 6) = 0.893$
Answer: (a) 0.0668 (b) 0.107 (c) 0.893

4 Arc length and sector area (All boards)

A sector of a circle has arc length 4.2 cm and area 14.7 cm^2.
Find the radius of the circle and the angle of the sector, in radians.

Area = $\frac{1}{2}rs$: $14.7 = \frac{1}{2} \times r \times 4.2$

$r = \frac{2 \times 14.7}{4.2} = 7$ cm

$\theta = \frac{s}{r} = \frac{4.2}{7} = 0.6$ rad

Answer: $r = 7$ cm, $\theta = 0.6$ rad

5 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 5$, $y = 2t(t^2 - 1)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 1$, so $t = 0, 1, -1$
$t = 0$: $x = -5$
$t = \pm 1$: $x = -2$
Answer: (−5, 0) and (−2, 0)

6 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 5 kg particle moves on a smooth horizontal line. At time t s a single force of $(15 - 3t)$ N acts along the line. When t = 0 its speed is 1 m s^{-1}. Find (a) its speed when t = 8, (b) its greatest speed.

F = ma: $a = (-3t + 15) \div 5$
$v = \int a\,dt = 3t - 0.3t^2 + c$; v = 1 when t = 0
(a) v(8) = 5.8
(b) Greatest speed when a = 0: t = 5
v(5) = 8.5
Answer: (a) 5.8 m s^{-1} (b) 8.5 m s^{-1}

7 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where t = 3, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{t^2 + 1}{t^2 - 1}$

t = 3: point $(\frac{10}{3}, \frac{8}{3})$, gradient $\frac{5}{4}$

Normal gradient $= -1 \div \frac{5}{4} = -\frac{4}{5}$, so normal: $12x + 15y - 80 = 0$

Answer: $\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$; $12x + 15y - 80 = 0$

8 Factor theorem and remainder theorem (All boards)

$f(x) = x^3 + ax^2 + bx + 48$, where a and b are constants.
$(x - 4)$ is a factor of f(x). The remainder when f(x) is divided by $(x + 1)$ is 60.
(a) Find a and b. (b) Solve f(x) = 0.

f(4) = 0: $64 + 16a + 4b + 48 = 0$
f(−1) = 60: $-1 + a - b + 48 = 60$
Solving simultaneously: a = −3, b = −16
$f(x) = x^3 - 3x^2 - 16x + 48 = (x - 4)(x + 4)(x - 3)$
Answer: (a) a = −3, b = −16 (b) x = −4, 3, 4

9 Sigma notation (All boards)

Given that $\sum_{r=1}^{n} (r - 2) = 119$, find the value of n.

$\sum_{r=1}^{n} (r - 2) = \frac{n(n + 1)}{2} - 2n$
$\frac{1}{2}n^2 - \frac{3}{2}n = 119$
$n^2 - 3n - 238 = 0$
Solve and reject the negative root: n = 17
Answer: 17

10 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1000 kg car tows a 300 kg trailer by a light tow-bar on a horizontal road. Driving force 610 N; resistances 200 N (car), 150 N (trailer). Find (a) the acceleration and the tow-bar tension. At 15 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $610 - 200 - 150 = 1300\,a$, so a = 0.2
Trailer: $T - 150 = 300\,a$, so $T = 150 + 300 \times 0.2 = 210$
(b) After the break the trailer decelerates at $150 \div 300 = 0.5$ m s^{-2}
$0 = 15^2 - 2 \times 0.5 \times s$, so s = 225
Answer: (a) 0.2 m s^{-2}, 210 N (b) 225 m