Day 51 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arrangements with restrictions (OCR, MEI, Eduqas, CIE)

How many arrangements of the letters of PAINTER have A and R not next to each other?

All arrangements = 7! = 5040
A and R together: 2 × 6! = 1440
Not together = 5040 − 1440 = 3600
Answer: 3600

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(-3 + 2x)^7$.

Term in x^4: ${}^7C_4 \times (-3)^3 \times (2x)^4$
$= 35 \times -27 \times 16$
Answer: −15120

3 Smallest whole number solving an exponential inequality (All boards)

£20000 is invested at 4% compound interest per year.
Find the smallest whole number of years for the value to exceed £60000.

$20000 \times 1.04^n > 60000$, so $1.04^n > 3$

$n \ln 1.04 > \ln 3$, so $n > \frac{\ln 3}{\ln 1.04} = 28.01$

$n = 29$
Answer: n = 29 years

4 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg box is pulled from rest along rough horizontal ground by a horizontal force of 20.8 N and moves 9.6 m in 8 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $9.6 = \frac{1}{2}a \times 8^2$, so $a = 0.3$
(a) $P - \mu 4g = 4a$: $20.8 - 39.2\mu = 1.2$, so $\mu = 0.5$
(b) $v = at = 0.3 \times 8 = 2.4$; after removal deceleration = $\mu g = 4.9$
$0 = 2.4^2 - 2 \times 4.9 \times s$, $s = 0.588$
Answer: (a) μ = 0.5 (b) 0.588 m

5 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean μ and standard deviation 12.
Given that P(X > 75) = 0.0668, find μ to 3 significant figures.

P(Z > z) = 0.0668, so z = 1.50

$\frac{75 - \mu}{12} = 1.50$

$\mu = 75 - (1.50)(12) = 57.0$
Answer: μ = 57.0

6 Partial fractions: repeated factor (All boards)

Express $\frac{-2x^2 - 10x - 13}{(x + 3)(x + 2)^2}$ in the form $\frac{A}{(x + 3)} + \frac{B}{(x + 2)} + \frac{C}{(x + 2)^2}$, and find A, B and C.

$-2x^2 - 10x - 13 \equiv A(x + 2)^2 + B(x + 3)(x + 2) + C(x + 3)$
$x = -3$: $-1 = A(-1)^2$ so $A = -1$
$x = -2$: $-1 = C(1)$ so $C = -1$
Coefficient of x^2: $-2 = A + B$ so $B = -1$
Answer: A = −1, B = −1, C = −1

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7 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (1 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (3 kg), hanging 1.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $3g - T = 3a$
A: $T = 1a$
Adding: $3g = 4a$, so $a = 7.35$
(a) $T = 1 \times 7.35 = 7.35$
(b) $v^2 = 0 + 2 \times 7.35 \times 1.5 = 22.05$
Answer: (a) 7.35 N (b) 4.70 m s^{-1}

8 Separable differential equations (All boards)

Solve the differential equation

$y\frac{dy}{dx} = 3x$,

given that $y = 3$ when $x = 0$ ($y > 0$). Give y in terms of x.

Separate: $\int y\,dy = \int 3x\,dx$, so $\frac{y^2}{2} = \frac{3}{2}x^2 + C$

$x = 0$, $y = 3$ gives $C = \frac{9}{2}$

$y^2 = 3x^2 + 9$
Answer: $y = \sqrt{3x^2 + 9}$

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x - 3y = -2$ meets the circle $(x - 2)^2 + (y + 2)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre $(2, -2)$, radius 6. Distance from centre to line:

$d = \frac{|16|}{5} = \frac{16}{5}$

Half-chord$^2 = 36 - (\frac{16}{5})^2 = \frac{644}{25}$

$AB = 2\sqrt{\frac{644}{25}}$

Answer: $\frac{20\sqrt{161}}{25}$

10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 + 2xy + y^2 = 18$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{4x + 2y}{2x + 2y}$
Horizontal tangent: $4x + 2y = 0$, so $y = -2x$
Substitute: $2x^2 = 18$, so $x^2 = 9$ and $x = \pm 3$
Points: $(3, -6)$ and $(-3, 6)$
Answer: (3, −6) and (−3, 6)