Day 50 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean windspeed (knots) in the same month are:
Perth: mean 13.5, standard deviation 1.8;
Jacksonville: mean 7.5, standard deviation 1.7.
Make two comparisons.

Means: Perth has the higher mean (13.5 against 7.5), so on average the mean windspeed is greater at Perth
Standard deviations: Perth is larger at Perth (1.8 against 1.7)
so the readings at Perth are more variable and those at Jacksonville are more consistent
Answer: Perth higher on average; Perth more variable

2 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (\mathbf{i} - 7\mathbf{j})$ N, $F_2 = (-6\mathbf{i} - 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 2 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$
i: $1 + (-6) + a = 0$, so $a = 5$
j: $-7 + (-8) + b = 0$, so $b = 15$
(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-6^2 + -8^2)} = 10$ N
$a = 10 \div 2$
Answer: (a) a = 5, b = 15 (b) 5 m s^{-2}

3 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 2n^2 + 3n$.
(a) Find the first term and the common difference.
(b) Find u_{20}.

$u_1 = S_1 = 5$
$S_2 = 14$, so $u_2 = 9$
$d = u_2 - u_1 = 4$
$u_{20} = 5 + 19 \times 4$
Answer: (a) a = 5, d = 4 (b) 81

4 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 40.8 m ahead. After a reaction time of 0.4 s she brakes with deceleration 2 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.4 v, braking distance = $v^2 \div (2\times2)$
$0.4\,v + v^2 \div 4 = 40.8$, so $v^2 + 1.6\,v - 163.2 = 0$
$(v - 12)(v + 13.6) = 0$, so $v = 12$
(b) Braking time = $12 \div 2 = 6$ s; total = 0.4 + 6
Answer: (a) 12 m s^{-1} (b) 6.4 s

5 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{8}{17}$ and $\cos B = -\frac{12}{13}$.

Find the exact values of sin(A−B) and tan(A+B).

$\cos A = \frac{15}{17}$ (A acute), $\sin B = \frac{5}{13}$ (B obtuse), so $\tan A = \frac{8}{15}$, $\tan B = -\frac{5}{12}$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B = \frac{8}{17} \times \left(-\frac{12}{13}\right) - \frac{15}{17} \times \frac{5}{13} = -\frac{171}{221}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{8}{15} + \left(-\frac{5}{12}\right)}{1 - \frac{8}{15} \times \left(-\frac{5}{12}\right)} = \frac{21}{220}$$

Answer: $\sin(A-B) = -\frac{171}{221}$, $\tan(A+B) = \frac{21}{220}$

6 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 50 items.
Use linear interpolation to estimate the upper quartile.

Position = $0.75 \times 50 = 37.5$
Cumulative frequency before the class $50 \leq t < 60$ is 33
Estimate = $50 + \frac{37.5 - 33}{17} \times 10$
$= 52.6$
Answer: 52.6

7 Tangent through the origin (All boards)

The line through the origin is a tangent to the curve $y = x^2 + 3$.
Find the gradients of the two possible tangents and the points of contact.

Tangent at x = a: $y - (a^2 + 3) = 2a(x - a)$. Through (0, 0): $-a^2 - 3 = -2a^2$
So $a^2 = 3$ and $a = \pm\sqrt{3}$
Gradient $2a = \pm 2\sqrt{3}$
Answer: gradients $\pm 2\sqrt{3}$ at points $(\pm\sqrt{3}, 6)$

8 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 - 8x - 4y - 149 = 0$ passes through P(−8, −3).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (4, 2). Gradient of radius CP = $\frac{5}{12}$
Gradient of tangent = $-\frac{12}{5}$
$y + 3 = -\frac{12}{5}(x + 8)$: $12x + 5y = -111$
$y = 0$: $x = -\frac{37}{4}$
Answer: (a) $12x + 5y = -111$ (b) $\left(-\frac{37}{4}, 0\right)$

9 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(−1, −7) and B(2, −6).
Its centre lies on the line y = 3x + 1.
Find the equation of the circle.

Centre (h, 3h + 1) is equidistant from A and B.
$(h + 1)^2 + (3h + 8)^2 = (h - 2)^2 + (3h + 7)^2$
Solving gives h = −1, so centre (−1, −2)
$r^2 = (0)^2 + (-5)^2 = 25$
Answer: $(x + 1)^2 + (y + 2)^2 = 25$

10 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $-2x^2 + 2xy + 2y^2 - 3x + 3y = 0$.
Show that P(−3, 3) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(-4x + 2y - 3) + (2x + 4y + 3)\frac{dy}{dx} = 0$
At P(−3, 3): $15 + (9)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{5}{3}$
Tangent: $y - 3 = \left(-\frac{5}{3}\right)(x + 3)$, i.e. $5x + 3y + 6 = 0$
Answer: $5x + 3y + 6 = 0$