Day 5 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily maximum gust is modelled by N(20.6, 6^2).
(a) Find the probability that on a randomly chosen day the value exceeds 30 knots.
(b) Estimate the number of such days in a period of 62 days.

(a) $z = \frac{30 - 20.6}{6} = 1.57$

$P(Z > 1.57) = 0.0586$

(b) $62 \times 0.0586 = 3.63 \approx 4$

Answer: (a) 0.0586 (b) 4 days

2 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 12 kg is pushed at constant speed along rough horizontal ground, $\mu = \frac{1}{3}$, by a force of P N at angle θ below the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take g = 9.8.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: $R = 12g + P\sin\theta$

Horizontal: $P\cos\theta = \mu R = \frac{1}{3}(12g + P\sin\theta)$

$P(\frac{5}{13} - \frac{4}{13}) = \frac{1}{3} \times 12g$

$P = 510$, $R = 588$

Answer: P = 509.6 N, R = 588 N

3 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2\theta + 3\sin\theta - 3 = 0$
for $0° \le \theta \le 180°$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$: $2\sin^2\theta - 3\sin\theta + 1 = 0$

Solving the quadratic: $\sin\theta = \frac{1}{2}$ or $\sin\theta = 1$

$\sin\theta = \frac{1}{2}$: $\theta = 30°, 150°$

$\sin\theta = 1$: $\theta = 90°$

Answer: θ = 30°, 90°, 150°

4 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 2000 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(100 + 60) × V
2000 = 80 V, so V = 25
(b) Deceleration = V ÷ 25 = 25 ÷ 25

Answer: (a) V = 25 (b) 1 m s^{-2}

5 Inverse of a quadratic function (All boards)

$f(x) = x^2 - 4x + 2$, $x \ge 2$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = (x - 2)^2 - 2$

Let $y = f(x)$: $(x - 2)^2 = y + 2$

$x \ge 2$, so take the positive root: $x = 2 + \sqrt{y + 2}$

Answer: $f^{-1}(x) = 2 + \sqrt{x + 2}$, $x \ge -2$

6 Equations with sec x and tan x (All boards)

Solve $4\sec^2 x - 14\tan x - 12 = 0$
for $0° \le x \le 360°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $4\tan^2 x - 14\tan x - 8 = 0$

$\tan x = -\frac{1}{2}$ or $\tan x = 4$

$\tan x = -\frac{1}{2}$: $x = 153.4°, 333.4°$

$\tan x = 4$: $x = 76.0°, 256.0°$

Answer: x = 76.0°, 153.4°, 256.0°, 333.4°

7 Geometric series: smallest n (All boards)

A salary is £40000 in year 1 and increases by 4% each year.
In which year does the salary first exceed £80000?

Salary in year $n = 40000 \times 1.04^{n-1}$

$1.04^{n-1} > 2$

$n - 1 > \frac{\log 2}{\log 1.04} = 17.67$

$n = 19$

Answer: year 19

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{3}{64}x^2$ for $0 \le x \le 4$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \le x \le 4$.
(b) Find P(X > 2). (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{x^3}{64}$

(b) $P(X > 2) = 1 - F(2) = \frac{7}{8}$

(c) $F(m) = \frac{1}{2}$: $m^3 = \frac{4^3}{2}$, so $m = 4 \times (0.5)^{\frac{1}{3}}$

$m = 3.17$

Answer: (b) $\frac{7}{8}$ (c) 3.17

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x - 4y = 0$ meets the circle $(x + 4)^2 + (y - 2)^2 = 25$ at the points A and B. Find the exact length of AB.

Centre $(-4, 2)$, radius 5. Distance from centre to line:

$d = \frac{|-20|}{5} = 4$

$\text{Half-chord}^2 = 25 - (4)^2 = 9$

$AB = 2\sqrt{9}$

Answer: 6

10 Differentiating sec, cosec, tan and cot (All boards)

$y = \text{cosec}(3x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{12}$.

$\frac{dy}{dx} = -3\text{cosec}(3x)\cot(3x)$

At $x = \frac{\pi}{12}$ the angle $3x = \frac{\pi}{4}$, so the gradient is $-3\sqrt{2}$

Answer: $\frac{dy}{dx} = -3\text{cosec}(3x)\cot(3x)$; $-3\sqrt{2}$