Day 47 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Polynomial division (All boards)

Find the quotient and the remainder when $4x^3 + 18x^2 + 12x - 25$ is divided by $(2x + 5)$.

Quotient (matching leading terms in turn): $2x^2 + 4x - 4$

$(2x + 5)(2x^2 + 4x - 4) = 4x^3 + 18x^2 + 12x - 20$

Remainder $= 4x^3 + 18x^2 + 12x - 25 - (4x^3 + 18x^2 + 12x - 20) = -5$

Answer: quotient $2x^2 + 4x - 4$, remainder -5

2 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 18\cot x + 14 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 18\cot x + 18 = 0$

$\cot x = \frac{3}{2}$ or $\cot x = 3$

$\cot x = \frac{3}{2}$ means $\tan x = \frac{2}{3}$: $x = 33.7°, 213.7°$

$\cot x = 3$ means $\tan x = \frac{1}{3}$: $x = 18.4°, 198.4°$

Answer: $x = 18.4°, 33.7°, 198.4°, 213.7°$

3 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $F_1 = (23\mathbf{i} + 19\mathbf{j})$ N and F_2. Its acceleration is $(4\mathbf{i} + 3\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 5(4\mathbf{i} + 3\mathbf{j}) = 20\mathbf{i} + 15\mathbf{j}$

(a) $F_2 = (20\mathbf{i} + 15\mathbf{j}) - (23\mathbf{i} + 19\mathbf{j}) = -3\mathbf{i} - 4\mathbf{j}$

(b) $|F_2| = \sqrt{(-3^2 + -4^2)} = 5$

$\cos\theta = -3 \div 5$, so $\theta = 126.9°$

Answer: (a) $F_2 = (-3\mathbf{i} - 4\mathbf{j})$ N (b) 126.9°

4 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 4 + 3\cos\theta$, $y = -4 + 3\sin\theta$.
Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x - 4}{3}$, $\sin\theta = \frac{y + 4}{3}$

Use $\cos^2\theta + \sin^2\theta = 1$: $(x - 4)^2 + (y + 4)^2 = 9$

Answer: $(x - 4)^2 + (y + 4)^2 = 9$, a circle with centre $(4, -4)$, radius 3

5 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (0, 2, 0), (3, 2, 4) and (−1, −1, 1).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} -4 \\ -3 \\ -3 \end{pmatrix}$

$\text{BA}\cdot\text{BC} = 24$

$|\text{BA}| = \sqrt{25}$, $|\text{BC}| = \sqrt{34}$

$\cos\text{ABC} = \frac{24}{\sqrt{25} \times \sqrt{34}} = 0.8232$, so angle ABC = 34.6°

Answer: 34.6°

6 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 24t + 18)$ m s^{-1} at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 6 s.

(a) $v = 6(t - 1)(t - 3) = 0$, so $t = 1$ or $t = 3$

(b) Displacement $x = \int v\,dt = 2t^3 - 12t^2 + 18t$

$x(1) = 8$, $x(3) = 0$, $x(6) = 108$

Direction changes at $t = 1$ and $t = 3$, so distance $= |8| + |0 - 8| + |108 - 0|$

Answer: (a) t = 1, t = 3 (b) 124 m

7 Normal approximation to the Poisson (CIE)

$X \sim Po(18)$. Use a suitable approximation to find $P(X \geq 14)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(18, 18)$ approximately

Continuity correction: $P(Y > 13.5)$

$z = -1.06$, so probability = 0.856

Answer: 0.856

8 Tangent conditions (All boards)

The line $y = kx + 5$ is a tangent to the curve $y = x^2 - x + 21$.

(a) Find the possible values of k.

(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 - x + 21 = kx + 5$

$x^2 + (-1 - k)x + 16 = 0$

Tangent: discriminant = 0: $(-1 - k)^2 = 64$

$-1 - k = \pm 8$ so $k = 7$ or -9

$k = 7$: $x = \frac{k - (-1)}{2} = 4$, $y = 33$

Answer: (a) k = −9 or k = 7 (b) (4, 33)

9 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(23, p)$. H_0: $p = 0.3$ is tested against H_1: $p > 0.3$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.5$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \geq 12$

(b) Type I error = $P(\text{reject } H_0 \mid p = 0.3) = 0.0214$

(c) Type II error = $P(\text{not in critical region} \mid p = 0.5)$

$P(X \leq 11)$ when $p = 0.5$: 0.500

Answer: (a) X ≥ 12 (b) 0.0214 (c) 0.500

10 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, −1, −2) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} t + 4 \\ -2t - 3 \\ 2t + 4 \end{pmatrix}$

$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $9t + 18 = 0$, so $t = -2$

F = (3, 0, −2); $\mathbf{PF} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{5}$

Answer: F(3, 0, −2), distance $\sqrt{5}$