Day 46 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 29.4 m s^{-1} from a point h m above horizontal ground, and hits the ground 10 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 29.4, a = −9.8, t = 10
$-h = 29.4\times10 - 4.9\times10^2$, so h = 196
(b) At the highest point v = 0: $0 = 29.4^2 - 2\times9.8\times s$
s = 44.1 m above the point of projection
Greatest height above the ground = 196 + 44.1
Answer: (a) 196 m (b) 240.1 m

2 Arithmetic sequences: finding a and d (All boards)

The 3rd term of an arithmetic sequence is 4 and the 8th term is 34.
Find the first term and common difference, and the sum of the first 20 terms.

$u_8 - u_3 = 5\,d = 34 - 4$ so d = 6
a + 2 d = 4 so a = −8
$S_{20} = \frac{20}{2}(2\times(-8) + 19\times6)$
Answer: a = −8, d = 6, S_{20} = 980

3 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

|a| = 3, |b| = 8 and the angle between a and b is 90°.
Find the exact values of a·b and |a + b|.

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos 90° = 3\times8\times0 = 0$
$|\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}|^2 + 2\mathbf{a}\cdot\mathbf{b} + |\mathbf{b}|^2 = 9 + 0 + 64 = 73$
$|\mathbf{a}+\mathbf{b}| = \sqrt{73}$
Answer: $\mathbf{a}\cdot\mathbf{b} = 0$, $|\mathbf{a}+\mathbf{b}| = \sqrt{73}$

4 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The life of a battery is N(71, 5^2). A battery is classed as 'special' if its life is less than 62.
10 batterys are chosen at random. Find the probability that exactly 4 are special (3 s.f.).

P(special) = P(X < 62) = P(Z < −1.80) = 0.0359
Y ~ B(10, 0.0359)
$P(Y = 4) = C(10, 4)(0.0359)^4(0.964)^6 = 0.000281$
Answer: 0.000281

5 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (9 kg) and B (1 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: 9 g − T = 9 a; B: T − 1 g = 1 a
a = 8 g ÷ 10 = 7.84, so T = 1(g + a) = 17.64
When A lands, B has risen 2 m with $v^2 = 2\times7.84\times2 = 31.36$
B then moves freely under gravity: extra height = $v^2 \div (2\times9.8) = 1.6$
Total = 2 + 1.6
Answer: (a) 17.64 N (b) 3.6 m

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6 Asymptotes after transformation (All boards)

The curve y = f(x), where f(x) = $\frac{4}{x+3} + 1$, is translated by $\begin{pmatrix}1\\1\end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

Asymptotes of y = f(x): x = −3, y = 1

Translate by $\begin{pmatrix}1\\1\end{pmatrix}$: x = −2, y = 2

New curve: y = $\frac{4}{x+2} + 2$

y = 0: $\frac{4}{x+2} = -2$, so x + 2 = −2

Answer: x = −2, y = 2; crosses x-axis at (−4, 0)

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = Ax^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

n = $\frac{8-6}{3-2} = 2$

At (2, 6): ln A = 6 − 2 × 2 = 2, so $A = e^2$

Answer: n = 2, A = e^2

8 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $x^2 - y^2 = 22$.

Assume $x^2 - y^2 = (x - y)(x + y) = 22$.
x − y and x + y are both odd or both even (they differ by 2y).
Both odd: the product is odd, but 22 is even.
Both even: the product is a multiple of 4, but 22 is not.
Contradiction either way.
Answer: no such integers exist

9 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(30, 0.25).
Find the smallest value of c such that P(X ≥ c) < 0.05.

P(X ≥ 13) = 1 − P(X ≤ 12) = 0.0216
P(X ≥ 12) = 1 − P(X ≤ 11) = 0.0507
0.0216 < 0.05 but 0.0507 > 0.05
Answer: c = 13

10 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
5sin 2θ + 3cos θ = 7.5
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with 0 < θ < 0.8, to 3 s.f.

sin 2θ ≈ 2θ, cos θ ≈ $1 - \frac{1}{2}\theta^2$

$5 \times 2\theta + 3(1 - \frac{1}{2}\theta^2) = 7.5$

Multiply by 2 and rearrange: $3\theta^2 - 20\theta + 9 = 0$

θ = $\frac{20 \pm \sqrt{292}}{6}$, so θ = 0.485

Answer: a = 3, b = −20, c = 9; θ ≈ 0.485