Day 44 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 4 kg is acted on by forces $\mathbf{F}_1 = (-12\mathbf{i} + 19\mathbf{j})$ N and $\mathbf{F}_2$. Its acceleration is $(3\mathbf{i} + 3\mathbf{j})$ m s^{-2}. Find (a) $\mathbf{F}_2$, (b) the angle between $\mathbf{F}_2$ and $\mathbf{i}$, to 1 d.p.

$\mathbf{F} = m\mathbf{a}$: $\mathbf{F}_1 + \mathbf{F}_2 = 4(3\mathbf{i} + 3\mathbf{j}) = 12\mathbf{i} + 12\mathbf{j}$

(a) $\mathbf{F}_2 = (12\mathbf{i} + 12\mathbf{j}) - (-12\mathbf{i} + 19\mathbf{j}) = 24\mathbf{i} - 7\mathbf{j}$

(b) $|\mathbf{F}_2| = \sqrt{(24^2 + -7^2)} = 25$

$\cos\theta = 24 \div 25$, so $\theta = 16.3°$

Answer: (a) $\mathbf{F}_2 = (24\mathbf{i} - 7\mathbf{j})$ N (b) 16.3°

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Camborne: mean 23.5, standard deviation 4.1;
Leeming: mean 16.5, standard deviation 1.6.
Make two comparisons.

Means: Camborne has the higher mean (23.5 against 16.5), so on average the mean visibility is greater at Camborne

Standard deviations: Camborne is larger at Camborne (4.1 against 1.6)

so the readings at Camborne are more variable and those at Leeming are more consistent

Answer: Camborne higher on average; Camborne more variable

3 Perpendicular bisector and area (All boards)

A is (−4, −5) and B is (−6, 5).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (−5, 0); gradient AB = −5

Perpendicular gradient = $\frac{1}{5}$

Equation: $x - 5y = -5$

$y = 0$: P = (−5, 0)

Area = ½ |(−6 − (−4))(0 − (−5)) − (−5 − (−4))(5 − (−5))|

Answer: (a) $x - 5y = -5$ (b) 0

4 Logarithmic equations (All boards)

Solve $\log_{10}(x + 2) + \log_{10}(x - 7) = 1$.

Combine: $\log_{10}[(x + 2)(x - 7)] = 1$, so $(x + 2)(x - 7) = 10^1 = 10$

$x^2 - 5x - 24 = 0$: $(x - 8)(x + 3) = 0$

$x = 8$ or $x = -3$; $x = -3$ makes $x - 7$ negative, so it is rejected

Answer: $x = 8$

5 Foot of a perpendicular (All boards)

The line l has equation $x + y = -6$.
(a) Find the coordinates of the foot of the perpendicular from P(6, −4) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = −1, so perpendicular gradient = 1

Perpendicular through P: $x - y = 10$

Solve with l: $x = 2$, $y = -8$

Distance$^2 = (6 - 2)^2 + (-4 - (-8))^2 = 32$

Answer: (a) (2, −8) (b) $4\sqrt{2}$

6 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 10 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, $\sin\beta = \frac{12}{13}$, $\cos\beta = \frac{5}{13}$

Horizontal: $T_1\cos\alpha = T_2\cos\beta$, so $T_2 = 2.08\,T_1$

Vertical: $T_1\sin\alpha + T_2\sin\beta = 10\,g = 98$

$T_1(0.6 + 1.92) = 98$

$T_1 = 38.9$, $T_2 = 80.9$

Answer: T_1 = 38.9 N, T_2 = 80.9 N

7 Normal approximation to the binomial (All boards)

X ~ B(100, 0.3). Use a normal approximation to estimate P(X ≥ 35) to 3 significant figures.

np = 30, npq = 21, so Y ~ N(30, 21)

With continuity correction: P(Y > 34.5)

$z = \frac{34.5 - 30}{4.58} = 0.982$, P(Z > 0.982) = 0.163

Answer: 0.163

8 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{12}{13}$ and $\cos B = -\frac{5}{13}$.

Find the exact values of tan(A+B) and tan(A−B).

$\cos A = \frac{5}{13}$ (A acute), $\sin B = \frac{12}{13}$ (B obtuse), so $\tan A = \frac{12}{5}$, $\tan B = -\frac{12}{5}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} = \frac{\frac{12}{5} + (-\frac{12}{5})}{1 - \frac{12}{5}\times(-\frac{12}{5})} = 0$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} = \frac{\frac{12}{5} - (-\frac{12}{5})}{1 + \frac{12}{5}\times(-\frac{12}{5})} = -\frac{120}{119}$$

Answer: tan(A+B) = 0, tan(A−B) = $-\frac{120}{119}$

9 Differential equations in context (All boards)

A drink at 88°C is left in a room at 18°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 18)$.

After 15 minutes the temperature is 58°C. Find k, then the time for it to reach 28°C, to 3 s.f.

$\int \frac{1}{\theta - 18}\,d\theta = \int -k\,dt$, so $\ln(\theta - 18) = -kt + C$

t = 0: $\theta - 18 = 70$, so $\theta - 18 = 70e^{-kt}$

t = 15: $40 = 70e^{-15k}$, so $k = \frac{\ln(\frac{7}{4})}{15} = 0.0373$

$10 = 70e^{-kt}$: $t = \frac{\ln(7)}{k} = 52.2$ minutes

Answer: k = 0.0373, t = 52.2 minutes

10 Partial fractions: repeated factor (All boards)

Express $\frac{-x^2 + 25x - 9}{(x+4)(x-1)^2}$ in the form $\frac{A}{(x+4)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$, and find A, B and C.

$-x^2 + 25x - 9 \equiv A(x-1)^2 + B(x+4)(x-1) + C(x+4)$

x = −4: $-125 = A(-5)^2$ so A = −5

x = 1: 15 = C(5) so C = 3

Coefficient of x^2: −1 = A + B so B = 4

Answer: A = −5, B = 4, C = 3