**Day 43** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

$l_1$: $\mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$

$l_2$: $\mathbf{r} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$

Determine whether $l_1$ and $l_2$ intersect. If they do, find the point of intersection.

Equate components: $2 + 2\lambda = -2 + 0\mu$; $6 + 0\lambda = 6 - 3\mu$; $-9 - 3\lambda = -3 + 2\mu$
Solving: $\lambda = -2$, $\mu = 0$, and all three equations are satisfied
Point of intersection = $(-2, 6, -3)$
**Answer: Intersect at $(-2, 6, -3)$**

## 2 Binomial expansion: a coefficient (All boards)

Find the coefficient of $x^4$ in the expansion of $(2 + 3x)^8$.

Term in $x^4$: ${}^8C_4 \times 2^4 \times (3x)^4$
$= 70 \times 16 \times 81$
**Answer: 90720**

## 3 Logarithmic equations (All boards)

Solve $2\log_3 x - \log_3(x + 54) = 2$.

$\log_3 \frac{x^2}{x + 54} = 2$, so $\frac{x^2}{x + 54} = 3^2 = 9$
$x^2 = 9(x + 54)$: $x^2 - 9x - 486 = 0$
$x = 27$ or $x = -18$; $\log_3 x$ needs $x > 0$, so reject the negative root
**Answer: $x = 27$**

## 4 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $16^{3x + 2} = 8^{3x - 3}$.

Write both sides as powers of 2.
$(2^4)^{3x + 2} = (2^3)^{3x - 3}$
$4(3x + 2) = 3(3x - 3)$
$12x + 8 = 9x - 9$
$3x = -17$
**Answer: $x = -\frac{17}{3}$**

## 5 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 10$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{10}{x_n})$,

and use it with $x_0 = 3$ to find $x_2$ as an approximation to $\sqrt{10}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 10}{2x_n} = \frac{1}{2}(x_n + \frac{10}{x_n})$

$x_1 = \frac{1}{2}(3 + \frac{10}{3}) = \frac{19}{6}$
$x_2 = 3.1623$
**Answer: $x_2 = 3.1623$**

## 6 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a seed germinates is 0.4. In a sample of 24, the number of successes is 6. Test at the 5% level whether p has decreased.

$H_0$: p = 0.4, $H_1$: p < 0.4; X ~ B(24, 0.4)

$P(X \leq 6) = 0.0960$

0.0960 > 0.05

Do not reject $H_0$: insufficient evidence of a change in p.

**Answer: p-value 0.0960; do not reject $H_0$**

## 7 Velocity, rest and total distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves on the x-axis with velocity $v = (6t^2 - 24t + 18)$ m s$^{-1}$ at time t s, $t \geq 0$, and starts at O. Find (a) the times when P is at instantaneous rest, (b) the total distance P travels in the first 4 s.

(a) $v = 6(t - 1)(t - 3) = 0$, so t = 1 or t = 3

(b) Displacement $x = \int v\,dt = 2t^3 - 12t^2 + 18t$

x(1) = 8, x(3) = 0, x(4) = 8

Direction changes at t = 1 and t = 3, so distance = |8| + |0 − 8| + |8 − 0|

**Answer: (a) t = 1, t = 3 (b) 24 m**

## 8 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is normally distributed with mean 54 and standard deviation 12. Find (a) P(X > 73.2) (b) P(73.2 < X < 92.4), each to 3 s.f.

(a) $z = \frac{73.2 - 54}{12} = 1.60$, $P(Z > 1.60) = 0.0548$

(b) $z_1 = 1.60$, $z_2 = \frac{92.4 - 54}{12} = 3.20$

$P(73.2 < X < 92.4) = \Phi(3.20) - \Phi(1.60) = 0.0541$

**Answer: (a) 0.0548 (b) 0.0541**

## 9 Binomial expansion: approximating a square root (All boards)

(a) Expand $(1 - 2x)^{\frac{1}{2}}$ in ascending powers of $x$ up to $x^2$.

(b) Use x = 0.02 to estimate $\sqrt{96}$ to 4 decimal places.

$(1 + y)^{\frac{1}{2}} = 1 + \frac{1}{2}y - \frac{1}{8}y^2$, $y = -2x$

(a) $1 - x - \frac{1}{2}x^2$

$x = \frac{1}{50}$: $96 = 10^2(1 - \frac{1}{25})$ so $\sqrt{96} = 10 \times (1 - \frac{1}{25})^{\frac{1}{2}}$

$\approx 10 \times (1 - \frac{1}{50} - \frac{1}{5000}) = 9.798000$

**Answer: (a) $1 - x - \frac{1}{2}x^2$ (b) 9.7980**

## 10 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1200 kg car tows a 200 kg trailer by a light tow-bar on a horizontal road. Driving force 930 N; resistances 500 N (car), 150 N (trailer). Find (a) the acceleration and the tow-bar tension. At 12 m s$^{-1}$ the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: 930 − 500 − 150 = 1400 a, so a = 0.2

Trailer: T − 150 = 200 a, so T = 150 + 200×0.2 = 190

(b) After the break the trailer decelerates at 150 ÷ 200 = 0.75 m s$^{-2}$

$0 = 12^2 - 2 \times 0.75 \times s$, so s = 96

**Answer: (a) 0.2 m s$^{-2}$, 190 N (b) 96 m**