Day 42 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Independent and mutually exclusive events (All boards)

P(A) = 0.3 and P(A ∪ B) = 0.79.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) P(A ∪ B) = P(A) + P(B) − P(A)P(B), so 0.79 = 0.3 + P(B)(1 − 0.3)

P(B) = $\frac{0.79 - 0.3}{0.7} = 0.7$

(b) P(A ∩ B) = 0, so P(B) = 0.79 − 0.3 = 0.49

(c) P(A′ ∩ B) = P(A′)P(B) = 0.7 × 0.7 = 0.49

Answer: (a) 0.7 (b) 0.49 (c) 0.49

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(1 - 3x)^7$.

Term in x^2: ${}^7C_2 \times 1^5 \times (-3x)^2$

$= 21 \times 1 \times 9$

Answer: 189

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^4 (2\sqrt{x} + \frac{2}{\sqrt{x}})\,dx$.

Integrate: $[\frac{4}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}}]$

Substitute the limits and subtract: $\frac{40}{3}$

Answer: $\mathbf{\frac{40}{3}}$

4 Continuous random variable: finding k and E(X) (CIE)

The random variable X has probability density function f(x) = kx(6 − x) for 0 ≤ x ≤ 6, and 0 otherwise.
(a) Find k. (b) Find E(X). (c) Find P(X > 2).

(a) $\int_0^6 f(x)\,dx = 1$ gives 36 k = 1, so k = $\frac{1}{36}$

(b) E(X) = $\int x\,f(x)\,dx = 3$

(c) P(X > 2) = $\int_2^6 f(x)\,dx = \frac{20}{27}$

Answer: (a) k = $\mathbf{\frac{1}{36}}$ **(b) 3 (c)** $\mathbf{\frac{20}{27}}$

5 Smallest whole number solving an exponential inequality (All boards)

Find the least integer n for which $2^n > 10^7$.

n lg 2 > 7, so n > $\frac{7}{\lg 2} = 23.25$

Since n must be an integer, n = 24

Answer: n = 24

6 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 3 m, mass 10 kg) has its centre of mass at G, AG = 1 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 2.5 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: $2T = (10 + m)g$, so $T = \frac{1}{2}(10 + m)g$
Moments about A: $T\times3 = 10\,g\times1 + m\,g\times2.5$
$\frac{1}{2}(10 + m)\times3 = 10 + 2.5\,m \rightarrow 15 + 1.5\,m = 10 + 2.5\,m$
$m\,(2.5 - 1.5) = 15 - 10$, so $m = 5$
(b) $T = \frac{1}{2}(10 + 5)\times9.8$
Answer: (a) m = 5 (b) 73.5 N

7 Inverse of a quadratic function (All boards)

$f(x) = 3x^2 + 1$, $x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

$f(x) = 3x^2 + 1$

Let $y = f(x)$: $x^2 = \frac{y - 1}{3}$

$x \geq 0$, so take the positive root: $x = \sqrt{\frac{y - 1}{3}}$

Answer: $f^{-1}(x) = \sqrt{\frac{x - 1}{3}}$, $x \geq 1$

8 Friction: find μ then stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 12 kg box is pulled from rest along rough horizontal ground by a horizontal force of 54.24 N and moves 4.8 m in 4 s. Find (a) the coefficient of friction. The force is then removed; (b) find the further distance moved. Take g = 9.8. Give answers to 3 s.f. if not exact.

$s = \frac{1}{2}at^2$: $4.8 = \frac{1}{2}a\times4^2$, so $a = 0.6$
(a) $P - \mu 12\,g = 12\,a$: $54.24 - 117.6\,\mu = 7.2$, so $\mu = 0.4$
(b) $v = at = 0.6\times4 = 2.4$; after removal deceleration $= \mu g = 3.92$
$0 = 2.4^2 - 2\times3.92\times s$, $s = 0.735$
Answer: (a) μ = 0.4 (b) 0.735 m

9 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}12\\3\\-8\end{pmatrix} + \lambda\begin{pmatrix}-3\\-2\\3\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}6\\-2\\-2\end{pmatrix} + \mu\begin{pmatrix}-3\\0\\1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $12 - 3\lambda = 6 - 3\mu$; $3 - 2\lambda = -2 + 0\mu$; $-8 + 3\lambda = -2 + 1\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{7x + 1}{(1 - x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 + x)}$: A = 4, B = −3

(b) $4(1 - x)^{-1} = 4(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need |x| < 1 and |x| < 1

Answer: (a) $\frac{4}{(1 - x)} + \frac{-3}{(1 + x)}$ **(b)** $1 + 7x + x^2$ **(c)** $|x| < 1$