Day 41 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$; friction $F = \mu R = 0.1 \times 3g\cos\alpha$

B: $6g - T = 6a$

A: $T - 3g\sin\alpha - F = 3a$

Adding: $g(6 - 2.4 - 0.18) = 9a$

(a) $a = 3.724$; (b) $T = 6(g - a)$

Answer: (a) 3.724 m s^{-2} (b) 36.5 N

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2\theta + 3\sin\theta - 3 = 0$
for $0° \le \theta \le 360°$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$: $2\sin^2\theta - 3\sin\theta + 1 = 0$

Solving the quadratic: $\sin\theta = 1$ or $\sin\theta = \frac{1}{2}$

$\sin\theta = 1$: $\theta = 90°$

$\sin\theta = \frac{1}{2}$: $\theta = 30°, 150°$

Answer: θ = 30°, 90°, 150°

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 110 items in a list are numbered 1 to 110.
A systematic sample of 11 items is taken, starting with a random item among the first few.
The first item selected is number 8.
Find the sampling interval and the number of the last item selected.

Interval = 110 ÷ 11 = 10

Items selected: 8, 18, 28, ...

Last = 8 + (11 − 1) × 10 = 108

Answer: Interval 10, last item 108

4 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = t - 3$, $y = -2t^2 - 5$.
Find a Cartesian equation of the curve in the form $y = f(x)$.

From x: $t = x + 3$

$y = -2(x + 3)^2 - 5$

Expand.

Answer: $y = -2x^2 - 12x - 23$

5 Product rule: stationary points (All boards)

$y = x^2 \ln x$ for $x > 0$.
Find the exact coordinates of the stationary point and show that it is a minimum.

$\frac{dy}{dx} = 2x\ln x + x^2 \times \frac{1}{x} = x^1(2\ln x + 1)$

$\frac{dy}{dx} = 0$, $x > 0$: $\ln x = -\frac{1}{2}$, so $x = e^{-\frac{1}{2}}$

$y = x^2 \ln x = e^{-1} \times (-\frac{1}{2}) = -\frac{e^{-1}}{2}$

$\frac{d^2y}{dx^2} = 2 > 0$ at this point, so it is a minimum

Answer: $(e^{-\frac{1}{2}}, -\frac{e^{-1}}{2})$, minimum

6 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (10 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (4 kg), hanging 2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $4g - T = 4a$
A: $T = 10a$
Adding: $4g = 14a$, so $a = 2.8$
(a) $T = 10 \times 2.8 = 28$
(b) $v^2 = 0 + 2 \times 2.8 \times 2 = 11.2$
Answer: (a) 28 N (b) 3.35 m s^{-1}

7 Arithmetic series: finding n (All boards)

An arithmetic series has first term 5 and common difference 6.
The sum of the first n terms is 1776. Find n.

$\frac{n}{2}(2 \times 5 + (n - 1) \times 6) = 1776$

$6n^2 + 4n = 3552$

$6n^2 + 4n - 3552 = 0$

$(n - 24)(6n + 148) = 0$, so $n = 24$ or $n = -\frac{74}{3}$

n must be a positive integer, so $n = 24$
Answer: 24

8 Partial fractions and integration (All boards)

(a) Express $\frac{2x + 6}{(x + 1)^2}$ in the form $\frac{A}{x + 1} + \frac{B}{(x + 1)^2}$.

(b) Hence find the exact value of $\int_0^4 \frac{2x + 6}{(x + 1)^2}\,dx$.

$2x + 6 \equiv A(x + 1) + B$, so $A = 2$ and $B = 4$

$I = [2\ln(x + 1) - \frac{4}{x + 1}]_0^4$

$= 2\ln\frac{5}{1} + 4(1 - \frac{1}{5})$

$= \frac{10\ln 5 + 16}{5}$

Answer: (a) A = 2, B = 4; (b) $\frac{10\ln 5 + 16}{5}$

9 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 61 and standard deviation 5.
Find (a) P(X > 58.5) (b) P(58.5 < X < 68.5), each to 3 s.f.

(a) $z = \frac{58.5 - 61}{5} = -0.500$, $P(Z > -0.500) = 0.691$

(b) $z_1 = -0.500$, $z_2 = \frac{68.5 - 61}{5} = 1.50$

$P(58.5 < X < 68.5) = \Phi(1.50) - \Phi(-0.500) = 0.625$
Answer: (a) 0.691 (b) 0.625

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-2x + 3}{(1 - 2x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 + 2x)}$: A = 1, B = 2

(b) $1(1 - 2x)^{-1} = 1(1 + 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|2x| < 1$

Answer: (a) $\frac{1}{(1 - 2x)} + \frac{2}{(1 + 2x)}$ **(b)** $3 - 2x + 12x^2$ **(c)** $|x| < \frac{1}{2}$