Day 40 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 560 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(90 + 50) × V
560 = 70 V, so V = 8
(b) Deceleration = V ÷ 20 = 8 ÷ 20

Answer: (a) V = 8 (b) 0.4 m s^{-2}

2 Venn diagrams and conditional probability (All boards)

P(A) = 0.6, P(B) = 0.55 and P(A ∩ B) = 0.5.
Find
(a) P(A ∪ B)
(b) P(A | B′)
(c) P(B′ | A′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.6 + 0.55 - 0.5 = 0.65$

(b) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.6 - 0.5}{1 - 0.55} = \frac{2}{9}$

(c) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.65}{1 - 0.6} = 0.875$

Answer: (a) 0.65 (b) $\frac{2}{9}$ (c) 0.875

3 Chain rule and gradients (All boards)

$y = (-2x - 2)^{-2}$.
Find $\frac{dy}{dx}$ and the gradient of the curve at x = 1.

Let $u = -2x - 2$, so $y = u^{-2}$, $\frac{du}{dx} = -2$, $\frac{dy}{du} = -2u^{-3}$

$\frac{dy}{dx} = 4(-2x - 2)^{-3}$

At x = 1: u = −4, so gradient = $-\frac{1}{16}$

Answer: $\frac{dy}{dx} = 4(-2x - 2)^{-3}$; gradient = $-\frac{1}{16}$

4 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^3 in the expansion of $(1 - 2x)^7$.

Term in x^3: ${}^7C_3 \times 1^4 \times (-2x)^3$
$= 35 \times 1 \times -8$

Answer: −280

5 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.
Find the exact values of A and k.

ln y = ln A + kx, a straight line with gradient k and intercept ln A
Intercept ln A = 3, so $A = e^3$
Gradient $k = \frac{2 - 3}{2 - 0} = -\frac{1}{2}$

Answer: $A = e^3$, $k = -\frac{1}{2}$

6 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{3}{\sqrt{11}-3}+\frac{4}{\sqrt{11}+3}$ can be written as $p+q\sqrt{11}$, and find p and q.

Common denominator $(\sqrt{11}-3)(\sqrt{11}+3) = 11-9 = 2$

Numerator $= 3(\sqrt{11}+3)+4(\sqrt{11}-3)$

$= -3+7\sqrt{11}$

Divide by 2.

Answer: $p = -\frac{3}{2}$, $q = \frac{7}{2}$

7 Two balls meeting (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Ball A is dropped from rest 88.2 m above the ground. At the same instant ball B is projected upwards at 44.1 m s^{-1} from the ground directly below A. Find (a) the time when they collide, (b) the height of the collision. Take g = 9.8.

Let the time be t. Distance fallen by A: $\frac{1}{2}gt^2 = 4.9t^2$

Height of B above the ground: $44.1t - 4.9t^2$

They meet when the distances add up to 88.2: $4.9t^2 + (44.1t - 4.9t^2) = 88.2$

so $44.1t = 88.2$, $t = 2$

Height of B $= 44.1\times 2 - 4.9\times 2^2 = 68.6$

Answer: (a) 2 s (b) 68.6 m

8 Tangent conditions (All boards)

The line $y = k - x$ is a tangent to the curve $y = \frac{16}{x}$.

Find the possible values of k.

$\frac{16}{x} = k - x$

$16 = kx - x^2$, so $x^2 - kx + 16 = 0$

Tangent: discriminant $k^2 - 4\times 1\times 16 = 0$

$k^2 = 64$

Answer: $k = 8$ or $k = -8$

9 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 36, and after 3 hours h = 1.

Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + C$

$t = 0$: $C = 2\times 6 = 12$, so $2\sqrt{h} = 12 - kt$

$t = 3$, $h = 1$: $2 = 12 - 3k$, so $k = \frac{10}{3}$

Empty when $h = 0$: $kt = 12$, so $t = \frac{12}{\frac{10}{3}} = \frac{18}{5}$ hours

Answer: $\frac{18}{5}$ hours

10 Combining two data sets (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times in seconds of two groups: group X has n = 20, mean 50, sd 7;
group Y has n = 20, mean 44, sd 8.
Find the mean and standard deviation of all 40 values (3 s.f. for the sd).

$\Sigma x = 20\times 50 + 20\times 44 = 1880$, mean = 47

Σx^2 (A) $= 20(7^2 + 50^2) = 50980$

Σx^2 (B) $= 20(8^2 + 44^2) = 40000$

Variance $= \frac{90980}{40} - 47^2 = 65.5$, so sd = 8.09

Answer: mean 47, sd 8.09