Day 39 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 18 m s^{-1}. 8 s later car B leaves P from rest, in the same direction, with acceleration 2 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 8) seconds.
B: ½×2×s^2 = 1 s^2 A: 18(s + 8)
1 s^2 − 18 s − 144 = 0 so s = 24 (s > 0)
(b) PQ = 1×24^2 = 576
(c) v = 0 + at = 2×24 = 48
Answer: (a) 24 s (b) 576 m (c) 48 m s^{-1}

2 Foot of a perpendicular (All boards)

The line l has equation 2x + y = 1.
(a) Find the coordinates of the foot of the perpendicular from P(−2, −3) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = −2, so perpendicular gradient = $\frac{1}{2}$
Perpendicular through P: x − 2y = 4
Solve with l: x = $\frac{6}{5}$, y = $-\frac{7}{5}$
Distance^2 = $(-2-\frac{6}{5})^2 + (-3-(-\frac{7}{5}))^2 = \frac{64}{5}$
Answer: (a) $(\frac{6}{5}, -\frac{7}{5})$ (b) $\frac{8\sqrt{5}}{5}$

3 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^{\ln 2} (e^{2x} + 5e^x)\,dx$.

Integrate: $[\frac{1}{2}e^{2x} + 5e^x]$
Substitute the limits and subtract: $\frac{13}{2}$
Answer: $\frac{13}{2}$

4 Binomial expansion: rational powers (All boards)

Expand $(1-2x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with n = $\frac{1}{3}$, y = −2x
Coefficients of 1, x, x^2: 1, $-\frac{2}{3}$, $-\frac{4}{9}$
Valid when |2x| < 1, i.e. |x| < $\frac{1}{2}$
Answer: $1 - \frac{2}{3}x - \frac{4}{9}x^2$; $|x| < \frac{1}{2}$

5 Double angle equations (All boards)

Solve $\cos 2x - 3\cos x = -1$
for $0 \leq x \leq 2\pi$.

$\cos 2x \equiv 2\cos^2 x - 1$: $2\cos^2 x - 3\cos x = 0$

Solve the quadratic: $\cos x = 0$ or $\cos x = \frac{3}{2}$

$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos x = \frac{3}{2}$ is impossible

Answer: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

6 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 8x - 12y + 32 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P$(-1, 2)$ lies inside or outside the circle.

Complete the square: $(x - 4)^2 + (y - 6)^2 = 20$
Centre (4, 6), radius $2\sqrt{5}$
Distance^2 from centre to P $= (-5)^2 + (-4)^2 = 41$
Compare with $r^2 = 20$
Answer: (a) centre (4, 6), radius $2\sqrt{5}$ (b) P is outside the circle

7 Independent and mutually exclusive events (All boards)

P(A) = 0.5 and P(A ∪ B) = 0.65.
(a) Given that A and B are independent, find P(B).
(b) Given instead that A and B are mutually exclusive, find P(B).
(c) In the case in (a), find P(A′ ∩ B).

(a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$, so $0.65 = 0.5 + P(B)(1 - 0.5)$
$P(B) = \frac{0.65 - 0.5}{0.5} = 0.3$
(b) $P(A \cap B) = 0$, so $P(B) = 0.65 - 0.5 = 0.15$
(c) $P(A' \cap B) = P(A')P(B) = 0.5 \times 0.3 = 0.15$
Answer: (a) 0.3 (b) 0.15 (c) 0.15

8 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3(t - \sin t)$, $y = 3(1 - \cos t)$.
Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{2\pi}{3}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{\sin t}{1 - \cos t}$
$t = \frac{2\pi}{3}$: gradient $m = \frac{\sqrt{3}}{3}$, point $(\frac{4\pi - 3\sqrt{3}}{2}, \frac{9}{2})$
$c = y - mx = \frac{18 - 2\sqrt{3}\pi}{3}$
Answer: $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$; $y = \frac{\sqrt{3}}{3}x + \frac{18 - 2\sqrt{3}\pi}{3}$

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (15 m, 15 kg) rests with A on rough ground, μ = 0.4, and B against a smooth wall, B being 12 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(15^2 - 12^2)} = 9$ m from the wall
Vertical: R = (15 + 50)g. Limiting friction: F = μR = 0.4(15 + 50)g
Horizontal: S = F (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance x×9/15):
S×12 = 15 g×4.5 + 50 g×(9 x ÷ 15)
26×12 = 67.5 + 30 x (g cancels), so x = 8.15
Answer: 8.15 m

10 Three-set Venn diagram counts (All boards)

Of 46 students, 26 study Maths (M), 16 Physics (P) and 20 Chemistry (C).
10 study M and P, 10 study M and C, 7 study P and C, and 4 study all three.
One student is chosen at random. Given that a student studies exactly two subjects, find the probability that they study Physics.

Venn regions: M only 10, P only 3, C only 7,
MP only 6, MC only 6, PC only 3, all three 4, outside all 7
Exactly two subjects: 6 + 6 + 3 = 15
Of these, Physics: 6 + 3 = 9
Probability = $\frac{9}{15}$
Answer: $\frac{3}{5}$